Lecture#2, Example#1

$y = 2$ When $x = 1$ in the solution found in step 2 to find the value of C.

$$\frac{1}{2}\ln\left(\frac{1}{3}\right) = C$$

The above implicit solution can be rewritten in an explicit form as(Step#3 becomes)

$$\frac{1}{2}\ln\frac{|y-1|}{|y+1|} = \ln|x| + \frac{1}{2}\ln\left(\frac{1}{3}\right)$$

$$\ln\{\frac{|y-1|}{|y+1|}\}\text{^}(\frac{1}{2}) = \ln|x| + \ln(\frac{1}{3})\text{^}(\frac{1}{2})$$

$$\ln\frac{\sqrt{y-1}}{\sqrt{y+1}} = \ln|x| + \ln\left(\frac{1}{\sqrt{3}}\right)$$

$$\ln\frac{\sqrt{y-1}}{\sqrt{y+1}} = ln\left(\frac{x}{\sqrt{3}}\right)$$

Taking antilog on both sides

$$\frac{\sqrt{y-1}}{\sqrt{y+1}} = \frac{x}{\sqrt{3}}$$

Taking square on both sides

$$\frac{\sqrt{y-1}^{\,2}}{\sqrt{y+1}^{2}} = \frac{x^{\,2}}{\sqrt{3}^{2}}$$

$$\frac{(y-1)}{(y+1)} = \frac{x^2}{3}$$

$$3(y-1) = x^2(y+1)$$

$$3y - 3 = x^2y + x^2$$

$$3y - x^2y = 3 + x^2$$

$$y(3 - x^2) = 3 + x^2$$

$$y = \frac{3 + x^2}{3 - x^2}$$

Example#2, Lecture #2

$$\frac{dy}{dt} = 1 + \frac{1}{y^2}$$

Separation of variables;

$$dy = \left(1 + \frac{1}{y^2}\right) dt$$

Divided both sides by $1 + \frac{1}{y^2}$ we get,

$$\frac{1}{1 + \frac{1}{y^2}} dy = \frac{\left(1 + \frac{1}{y^2}\right) dt}{\left(1 + \frac{1}{y^2}\right)}$$

$$\frac{1}{1 + \frac{1}{y^2}} dy = dt$$

Integrating both sides, we get

$$\int \frac{1}{1 + \frac{1}{y^2}} dy = \int dt$$