Question 6

$\frac{dr}{d\theta} + (\sec\theta)\, r = \cos\theta$

Comparing the given differential equation with standard form of linear 1st order differential equation;

$\frac{dr}{d\theta} + P(\theta)r = Q(\theta)$

$\implies P(\theta) = \sec\theta$ and $Q(\theta) = \cos\theta$

Now the integrating factor;

$IF = e^{\int P(\theta)d\theta} = e^{\int \sec\theta d\theta} = e^{\ln|\sec\theta+\tan\theta|} \quad \because \int \sec\theta d\theta = \ln|\sec\theta + \tan\theta|$

$IF = \sec\theta + \tan\theta \qquad \because e^{\ln t} = t \quad \forall t \in \mathbb{R}^+$

$\therefore$its solution is given by;

(Integrating Factor)$\times$(Dependent variable) $= \int$ ((Integrating Factor) $\times\, RHS)\, d\theta$

$\implies (\sec\theta + \tan\theta)\, r = \int (\sec\theta + \tan\theta)\cos\theta d\theta$

$\implies (\sec\theta + \tan\theta)\, r = \int \left(\frac{1}{\cos\theta} + \frac{1}{\cos\theta}\sin\theta\right)\cos\theta d\theta$

$\implies (\sec\theta + \tan\theta)\, r = \int \left(\frac{1}{\cos\theta}\cos\theta + \frac{1}{\cos\theta}\sin\theta\cos\theta\right) d\theta$

$\implies (\sec\theta + \tan\theta)\, r = \int (1 + \sin\theta)\, d\theta = \int 1 d\theta + \int \sin\theta d\theta$

$\implies (\sec\theta + \tan\theta)\, r = \theta + (-\cos\theta) + K$,where K is the constant of integration.

$\implies \boxed{r = \frac{1}{\sec\theta+\tan\theta}(\theta - \cos\theta + K)}$, is the required solution.