Question-1

$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$

Comparing the given differential equation with standard linear equation;

$\frac{dy}{dx} + P(x)y = Q(x)$

$\implies P(x) = \frac{2x+1}{x}$ and $Q(x) = e^{-2x}$

Now the integrating factor; $I.F = e^{\int P(x)dx} = e^{\int \frac{2x+1}{x} dx}$

$I.F = e^{\int \left(\frac{2x}{x} + \frac{1}{x}\right)dx}$ $\qquad\qquad \because \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$I.F = e^{\int \left(2+\frac{1}{x}\right)dx} = e^{\left(\int 2dx + \int \frac{1}{x}dx\right)} = e^{\left(2\int dx + \int \frac{1}{x}dx\right)}$

$I.F \quad = e^{(2x+\ln x)} = e^{2x}e^{\ln x} = xe^{2x}$ $\qquad\qquad e^{\ln t} = t \quad \forall\, t \in \mathbb{R}^{+}$

$\therefore$the solution is given by the following formula;

(Integrating Factor)$\times$(Dependent variable) $= \int$ ((Integrating Factor) $\times$ RHS) dx

$\implies xe^{2x}y = \int xe^{2x}e^{-2x}dx = \int xdx$

$\implies xe^{2x}y = \frac{1}{2}x^2 + C$, where C is the constant of integration.

$\implies \boxed{y = \frac{1}{xe^{2x}}\left(\frac{1}{2}x^2 + C\right)}$, is the required solution.

Question-2

$\frac{dy}{dx} + 3y = 3x^2e^{-3x}$

Comparing the given differential equation with standard linear equation;

$\frac{dy}{dx} + P(x)y = Q(x)$

$\implies P(x) = 3$ and $Q(x) = 3x^2e^{-3x}$

Now the integrating factor; $I.F = e^{\int P(x)dx} = e^{\int 3dx} = e^{3\int dx} = e^{3x}$

$\therefore$the solution is given by the following formula;

(Integrating Factor)$\times$(Dependent variable) $= \int$ ((Integrating Factor) $\times$ RHS) dx

$\implies e^{3x}y = \int e^{3x}3x^2e^{-3x}dx$

$\implies e^{3x}y = \int 3x^2dx = 3\int x^2dx = 3\frac{1}{3}x^3 + C = x^3 + C$

$\implies \boxed{y = e^{-3x}\left(x^3 + C\right)}$, where C is the constant of integration.