$\frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y+x-3}$

$\implies \frac{dy}{dx} = \frac{y(x+2)-1(x+2)}{y(x-3)+1(x-3)}$

$\implies \frac{dy}{dx} = \frac{(x+2)(y-1)}{(x-3)(y+1)}$

Separating the variables;

$\implies \frac{(y+1)}{(y-1)}dy = \frac{(x+2)}{(x-3)}dx$

Integrating;

$\implies \int \left(\frac{y+1}{y-1}\right) dy = \int \left(\frac{x+2}{x-3}\right) dx$

$\implies \int \left(\frac{y+1-1+1}{y-1}\right) dy = \int \left(\frac{x+2-3+3}{x-3}\right) dx$ *(Note this step)*

$\implies \int \left(\frac{(y-1)+2}{y-1}\right) dy = \int \left(\frac{(x-3)+5}{x-3}\right) dx$

$\because \left\{\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}\right.$

$\implies \int \left(\frac{y-1}{y-1} + \frac{2}{y-1}\right) dy = \int \left(\frac{x-3}{x-3} + \frac{5}{x-3}\right) dx$

$\implies \int \left(1 + \frac{2}{y-1}\right) dy = \int \left(1 + \frac{5}{x-3}\right) dx$

$\implies \int dy + \int \frac{2}{y-1}dy = \int dx + \int \frac{5}{x-3}dx$

$\implies \int dy + 2\int \frac{1}{y-1}dy = \int dx + 5\int \frac{1}{x-3}dx$

$\therefore \begin{cases} \int dt = t \\ \int \frac{f'(x)}{f(x)}dx = \ln|f(x)|, \text{ here } f(x) = x-3 \text{ and } f'(x) = 1 \\ \int \frac{f'(y)}{f(y)}dy = \ln|f(y)|, \text{ here } f(y) = y-1 \text{ and } f'(y) = 1 \end{cases}$

$\implies \boxed{y + 2\ln(y-1) = x + 5\ln(x-3) + C}$