

The values of t , where the graph of the solution $x(t) = \frac{2}{3}\sqrt{10}e^{-t} \sin(3t + 4.391)$ will meet the positive t -axis are given as follows;

$$\begin{aligned} x(t) &= \frac{2}{3}\sqrt{10}e^{-t} \sin(3t + 4.391) = 0 \\ \implies \sin(3t + 4.391) &= 0 \quad \because \frac{2}{3}\sqrt{10}e^{-t} \neq 0 \\ \implies 3t + 4.391 &= n\pi, n \in \mathbb{Z} \\ \implies 3t &= n\pi - 4.391 \implies t = \frac{\pi}{3}n - \frac{4.391}{3} \\ \implies t &= (1.0472)n - 1.4637 \end{aligned}$$

Put $n = 2, 3, 4, 5, \dots$ (here it is to be noted that we did not put $n = 1$, as this will give negative value of t which is of no interest)

$$\begin{aligned} t_{\gamma_1} &= (1.0472)(2) - 1.4637 = 0.6307 \\ t_{\gamma_2} &= (1.0472)(3) - 1.4637 = 1.6779 \\ t_{\gamma_3} &= (1.0472)(4) - 1.4637 = 2.7251 \\ t_{\gamma_4} &= (1.0472)(5) - 1.4637 = 3.7723 \end{aligned}$$

These are the values of t_γ in the 2nd column of the table.

The values of t , where the graph of the solution $x(t) = \frac{2}{3}\sqrt{10}e^{-t} \sin(3t + 4.391)$ will meet the graph of $\frac{2}{3}\sqrt{10}e^{-t}$ are given as follows;

$$\begin{aligned} x(t) &= \frac{2}{3}\sqrt{10}e^{-t} \sin(3t + 4.391) = \frac{2}{3}\sqrt{10}e^{-t} \\ \implies \sin(3t + 4.391) &= 1 \\ \implies 3t + 4.391 &= \text{odd multiples of } \frac{\pi}{2} \\ \implies 3t + 4.391 &= (2k + 1)\frac{\pi}{2}, k \in \mathbb{Z} \\ \implies 3t &= (2k + 1)\frac{\pi}{2} - 4.391 = 3.1416k - 2.8202 \\ \implies t &= \frac{3.1416k}{3} - \frac{2.8202}{3} \\ \implies t &= 1.0472k - 0.94007 \end{aligned}$$

Put $n = 1, 2, 3, 4, 5, \dots$

$$\begin{aligned} t_{\gamma_1}^* &= 1.0472(1) - 0.94007 = 0.10713 \\ t_{\gamma_2}^* &= 1.0472(2) - 0.94007 = 1.1543 \\ t_{\gamma_3}^* &= 1.0472(3) - 0.94007 = 2.2015 \\ t_{\gamma_4}^* &= 1.0472(4) - 0.94007 = 3.2487 \\ t_{\gamma_5}^* &= 1.0472(5) - 0.94007 = 4.2959 \end{aligned}$$

These are the values of t_γ^* in the 3rd column of the table.

