Since we get the final solution as;

$\frac{1}{2}\ln\left|\frac{y-1}{y+1}\right| = \ln|x| + C - - - (1)$

And the constant of integration is obtained as;

$\frac{1}{2}\ln\left(\frac{1}{3}\right) = C - - -$Put this in (1)

$\frac{1}{2}\ln\left|\frac{y-1}{y+1}\right| = \ln|x| + \frac{1}{2}\ln\left(\frac{1}{3}\right)$

$\{\because m\ln n = \ln n^m$ applying this just above on LHS and on the 2nd term of RHS.

$\Longrightarrow \ln\left|\frac{y-1}{y+1}\right|^{\frac{1}{2}} = \ln|x| + \ln\left(\frac{1}{3}\right)^{\frac{1}{2}}$

$\{\because \ln m + \ln n = \ln(mn)$ applying this just above on RHS

$\Longrightarrow \ln\left|\frac{y-1}{y+1}\right|^{\frac{1}{2}} = \ln\left[|x|\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]$

Taking anti-log on both sides,

$\Longrightarrow e^{\ln\left|\frac{y-1}{y+1}\right|^{\frac{1}{2}}} = e^{\ln\left[|x|\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]}$

$\{\because e^{\ln z} = z$ applying this just above on both sides

$\left|\frac{y-1}{y+1}\right|^{\frac{1}{2}} = \left[|x|\left(\frac{1}{3}\right)^{\frac{1}{2}}\right]$

Taking square on both sides, we get

$\frac{y-1}{y+1} = x^2\left(\frac{1}{3}\right)$

$\Longrightarrow \frac{y-1}{y+1} = \frac{x^2}{3}$

Applying the property of Componendo and Dividendo on both sides just above i.e. if $\frac{a}{b} = \frac{c}{d}$ then $\Longrightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

$\frac{(y-1)+(y+1)}{(y-1)-(y+1)} = \frac{x^2+3}{x^2-3}$

$\Longrightarrow \frac{2y}{-2} = \frac{x^2+3}{x^2-3}$

$\Longrightarrow -y = \frac{x^2+3}{x^2-3}$

$\Longrightarrow y = -\frac{x^2+3}{x^2-3}$

$\Longrightarrow y = \frac{x^2+3}{-(x^2-3)}$

$\Longrightarrow \boxed{y = \frac{3+x^2}{3-x^2}}$