$y(1+2xy)dx + x(1-2xy)dy = 0 - - - - - (1)$

Put $u = 2xy \Longrightarrow y = \frac{u}{2x} - - - -(a)$

$$\because \begin{cases} \frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2} \\ \therefore \frac{d}{dx} = \frac{d}{dx}\left(\frac{u}{2x}\right) \\ \frac{d}{dx}y = \frac{2x\frac{du}{dx} - u\frac{d}{dx}2x}{(2x)^2} \\ \frac{dy}{dx} = \frac{2x\frac{du}{dx} - u(1)}{(2x)^2} = \frac{1}{4x^2}\left(2x\frac{du}{dx} - 2u\right) \\ \Longrightarrow \frac{dy}{dx} = \frac{1}{2x^2}\left(\frac{xdu - udx}{dx}\right) \\ dy = \frac{xdu - udx}{2x^2} - - - -(b) \end{cases}$$

Using (a) and (b) in (1);

$\frac{u}{2x}(1+2x\frac{u}{2x})dx + x(1-2x\frac{u}{2x})\left(\frac{xdu-udx}{2x^2}\right) = 0$

$\frac{u}{2x}(1+u)dx + (1-u)\left(\frac{xdu-udx}{2x}\right) = 0$

$\Longrightarrow \frac{1}{2x}\left(u(1+u)dx + (1-u)(xdu - udx)\right) = 0$

$\Longrightarrow \left(u^2+u\right)dx + (xdu - udx - uxdu + u^2dx) = 0$

$\Longrightarrow u^2dx + udx + xdu - udx - uxdu + u^2dx = 0$

$\Longrightarrow 2u^2dx - x(u-1)du = 0$

$\Longrightarrow 2u^2dx = x(u-1)du$

Separating the varaibles;

$2\frac{1}{x}dx = \frac{u-1}{u^2}du$

Integrating;

$2\int \frac{1}{x}dx = \int \left(\frac{1}{u} - \frac{1}{u^2}\right)du$

$\Longrightarrow 2\ln x = \int \frac{1}{u}du - \int \frac{1}{u^2}du$

$\Longrightarrow 2\ln x = \ln u - \int u^{-2}du = \ln u - \left(-\frac{1}{u}\right) + C$

$\Longrightarrow 2\ln x = \ln u + \frac{1}{u} + C$