$y(4-x^2)\frac{1}{2}dy = (4+y^2)\frac{1}{2}dx$

Seperating the variables;

$\Longrightarrow y\frac{1}{(4+y^2)}dy = \frac{1}{(4-x^2)}dx$ 2 will cancel on both sides.

Integrating;

$\Longrightarrow \int y\frac{1}{(4+y^2)}dy = \int \frac{1}{(4-x^2)}dx$

Using the following formulas of integration on both sides,

i) $\begin{cases} \int \frac{f^{/}(y)}{f(y)}dy = \ln|f(y)| \\ \text{here } f(y) = 4+y^2 \Longrightarrow f^{/}(y) = 2y \end{cases}$ i.e for LHS.

ii) $\begin{cases} \int \frac{f^{/}(x)}{a^2-(f(x))^2}dx = \frac{1}{2}\ln\left|\frac{a-f(x)}{a+f(x)}\right| \\ \text{here } f(x) = x \Longrightarrow f^{/}(x) = 1,\ a = 2 \end{cases}$ i.e for RHS.

For LHS multiplying and dividing 2 to get the derivative of y^2 in numerator.

$\Longrightarrow \frac{1}{2}\int \frac{2y}{(4+y^2)}dy = \int \frac{1}{(4-x^2)}dx$

Now integrating by using above formulas,

$\Longrightarrow \frac{1}{2}\ln|4+y^2| = \frac{1}{2}\ln\left|\frac{2-x}{2+x}\right| + c$,where c is the constant of integration.

$\Longrightarrow \ln|4+y^2| = \ln\left|\frac{2-x}{2+x}\right| + 2c$

$\Longrightarrow \ln|4+y^2| = \ln\left|\frac{2-x}{2+x}\right| + \ln a$,where $2c = \ln a$ (say) is the constant of integration.

$\Longrightarrow \ln|4+y^2| = \ln\left(\left|\frac{2-x}{2+x}\right|a\right)$

Taking Anti-log;

$(4+y^2) = \left(\left|\frac{2-x}{2+x}\right|a\right) \Longrightarrow y^2 = \left|\frac{2-x}{2+x}\right|a - 4 \Longrightarrow$

$$y = \pm\sqrt{\left|\frac{2-x}{2+x}\right|a - 4}$$

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$\frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y+x-3}$

$\Longrightarrow \frac{dy}{dx} = \frac{y(x+2)-1(x+2)}{y(x-3)+1(x-3)}$

$\Longrightarrow \frac{dy}{dx} = \frac{(y-1)(x+2)}{(y+1)(x-3)}$

Seperating the variables;

$\Longrightarrow \frac{(y+1)}{(y-1)}dy = \frac{(x+2)}{(x-3)}dx$

$\Longrightarrow \frac{(y-1+1+1)}{(y-1)}dy = \frac{(x-3+2+3)}{(x-3)}dx$ (Please note this step for the numerator)

$\Longrightarrow \left(\frac{y-1}{y-1} + \frac{2}{y-1}\right)dy = \left(\frac{x-3}{x-3} + \frac{5}{(x-3)}\right)dx$

$\Longrightarrow \left(1 + \frac{2}{y-1}\right)dy = \left(1 + \frac{5}{x-3}\right)dx$

Integrating;

$\Longrightarrow \int \left(1 + \frac{2}{y-1}\right)dy = \int \left(1 + \frac{5}{x-3}\right)dx$

$\Longrightarrow \int 1dy + \int \frac{2}{y-1}dy = \int 1dx + \int \frac{5}{x-3}dx$

Using the following formulas of integration on both sides,

$$\text{i)}\begin{cases} \int \frac{f^{/}(y)}{f(y)}dy = \ln|f(y)| \\ \text{here } f(y) = y-1 \Longrightarrow f^{/}(y) = 1 \\ \int \frac{f^{/}(x)}{f(x)}dx = \ln|f(x)| \Longrightarrow f^{/}(x) = 1 \\ \int 1dy = y, \int 1dx = x \end{cases}$$

$\Longrightarrow y+2\ln(y-1) = x+5\ln(x-3)+c$, where c is the constant of integration.