Annihilator means to apply or choose some differential operator in such a way that it vanishes(becomes zero).

For example a constant is vanished on differentiating i.e. $\frac{d}{dx}a = 0$.So $\frac{d}{dx}$ is annihilator of any constant.Similarly, in case of x,if we differentiate it twice then we get zero i.e. $\frac{d}{dx}x = 1$, differentiate again, we have:

$\frac{d^2}{dx^2}x = \frac{d}{dx}\left(\frac{d}{dx}x\right) = \frac{d}{dx}1 = 0.$

$\therefore \frac{d^2}{dx^2}$ is annihilator of x.

Now if we denote $\frac{d}{dx} \equiv D$, $\frac{d^2}{dx^2} \equiv D^2$....etc.

Then for other functions, you can "*observe*" the followings;

$D^2 + 1$ is an annihilator of $\sin x$ and $\cos x$ i.e. $\left(D^2+1\right)\sin x = 0$ and $\left(D^2+1\right)\cos x = 0$

$\left(D^2+1\right)\sin x = D^2 \sin x + 1\sin x = \frac{d^2}{dx^2}\sin x + 1\sin x = \frac{d}{dx}\left(\frac{d}{dx}\sin x\right) + \sin x = \frac{d}{dx}(\cos x) + \sin x = -\sin x + \sin x = 0$

$\left(D^2+1\right)\cos x = D^2 \cos x + 1\cos x = \frac{d^2}{dx^2}\cos x + 1\cos x = \frac{d}{dx}\left(\frac{d}{dx}\cos x\right) + \cos x = \frac{d}{dx}(-\sin x) + \cos x = -\cos x + \cos x = 0$

Similarly for e^{ax},you can "*see*" $D-a$ is a annihilator i.e. $(D-a)\,e^{ax} = 0$

$(D-a)\,e^{ax} = De^{ax} - ae^{ax} = \frac{d}{dx}e^{ax} - ae^{ax} = ae^{ax} - ae^{ax} = 0$

And finally for x^2+3x-2, D^3 is annihilator i.e. $D^3\left(x^2+3x-2\right) = 0$

$D^3\left(x^2+3x-2\right) = \frac{d^3}{dx^3}\left(x^2+3x-2\right) = \frac{d^2}{dx^2}\left(\frac{d}{dx}\left(x^2+3x-2\right)\right) = \frac{d^2}{dx^2}(2x+3) = \frac{d}{dx}\left(\frac{d}{dx}(2x+3)\right) = \frac{d}{dx}(2) = 0$