

Lecture 34

Question 1: Marks: 2

You have 3 hats and 4 shirts. A hat and a shirt can be chosen in ways.

Solution:

Number of ways =12

Question 2: Marks: 3

How many different ways can you pick a team of 3 people from a group of 10.

Solution: 120.

Question 3: Marks: 3

How many different ways can you list 3 favorite desserts, in order, from a menu of 10.

Solution: 720

Question 4: Marks: 3

How many different arrangements are there of the letters of the word "**numbers**"?

Solution: 5,040

Question 5: Marks: 5

How many different 5-letter arrangements are there of the letters in the word

digit ?

Solution:

60

Lecture 35

Question 1: Marks: 2

In how many ways can 3 different books be arranged on a shelf?

Solution:

Number of ways = 6 ways

Question 2: Marks: 3

Construct a sample space S , if you toss a coin TWO times and observe the sequence of heads (H) and tails (T) those appears.

Solution:

Required sample space is

$$S = \{HH, HT, TH, TT\}$$

Question 3: Marks: 5

A teacher must choose five monitors from a class of 12 students. How many different ways can the teacher choose the monitors?

Solution: 792

Lecture 36

Question 1: Marks: 2

Choose a number at random from 1 to 5. What is the probability of each outcome?

Solution: Probabilities: $P(1) = 1/5$, $P(2) = 1/5$, $P(3) = 1/5$, $P(4) = 1/5$, $P(5) = 1/5$

Question 2: Marks: 3

If you toss a die and observe the number of dots that appears on top face then write the event that an odd number AND a number exceeding 4 occurs.

Solution:

A = event that an odd number occur = $\{1,3,5\}$

B = event that an even number occur = $\{2,4,6\}$

C = event that a number greater than 4 occurs = $\{5,6\}$

Thus the event that an even number and a number exceeding 4 occurs is

$$A \cap C = \{1,3,5\} \cap \{5,6\} = \{5\}$$

Question 3: Marks: 5

Two dice are rolled, find the probability that the sum is

a) equal to 1

b) equal to 5

c) less than 10

Solution:

Solution:

The sample space S of two dice is shown below.

$$S = \{ (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \}$$

Let E be the event "sum equal to 1". There are no outcomes which correspond to a sum equal to 1, hence

$$P(E) = n(E) / n(S) = 0 / 36 = 0$$

b) Three possible outcomes give a sum equal to 5: $E = \{(1,4),(2,3),(3,2), (4,1)\}$, hence.

$$P(E) = n(E) / n(S) = 4 / 36 = 1 / 9$$

c) All possible outcomes, $E = S$, give a sum less than 10, hence.

$$P(E) = n(E) / n(S) = 30 / 36$$

Lecture 37

Question 1: Marks: 3

Find the number **n** of distinct permutations that can be formed from the letters of the word: APPLE.

Solution: 60

Question 2: Marks: 5

A coin is tossed 10 times. What is the probability that exactly 6 heads will occur.

Solution:

1. Success = "A head is flipped on a single coin"
2. $p = 0.5$
3. $q = 0.5$
4. $n = 10$
5. $x = 6$

$$P(x=6) = {}_{10}C_6 * 0.5^6 * 0.5^4 = 210 * 0.015625 * 0.0625 = 0.205078125$$

Question 3: Marks: 5

Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

Solution:

This is a **binomial** distribution because there are only 2 outcomes (the patient dies, or does not).

The probability that 4 will recover = 0.0329595

Lecture 38

Question 1: Marks: 2

Write down formulas for the binomial probability mass function and cumulative binomial distribution.

Solution:

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$
$$B(x; n, p) = \sum_{y=0}^x \binom{n}{y} p^y (1-p)^{n-y}$$

Question 2: Marks: 5

Write down at least TWO properties of

1. Normal distribution.
2. Binomial Distribution.
3. Poisson distribution.

Lecture 39

Question 1: Marks: 3

Which type of distribution can better deal with the below mentioned cases?

- the number of deaths by car accidents
- birth defects and genetic mutations
- car accidents
- traffic flow and ideal gap distance
- number of typing errors on a page
- failure of a machine in one month

Question 2: Marks: 5

A company makes electric motors. The average number of defectives in 300 motors is $\mu = 3$. The probability an electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors?

Solution:

The probability of getting 5 defectives = 0.10082

Lecture 40

Question 1: Marks: 5

Write down the formula and properties of a Normal Distribution.

Question 2: Marks: 3

Find proportion of bags which have weight in excess of 600 g.

Mean = 595.5g

St. Dev = 3 g

(For $z=1.5$ probability is 0.4332)

Solution:

$z = (\text{Value } x - \text{Mean value}) / \text{Standard deviation}$

$z = (600 - 595.5) / 3 = 1.5$

Given probability corresponding to z value is 0.4332

Proportion of bags which have weight in excess of 600 g. = $0.5 - 0.4332 = 0.0668$

