Solution to Practice questions for Lecture No. 16 – 18

Solution 1:

Given function is

$$y = \frac{3x+2}{x+2}$$

We know that a function will be injective if for any two values $x_{1,}\ x_2$ from the domain of f,

$$f(x_1) = f(x_2)$$

Then

$$x_1 = x_2$$

Lets assume that for $x_{1,}\ x_2$, we have

$$\begin{aligned}
&\frac{3x_1+2}{x_1+2} = \frac{3x_2+2}{x_2+2} \\
&(3x_1+2)(x_2+2) = (3x_2+2)(x_1+2) \\
&3x_1x_2 + 6x_1 + 2x_2 + 4 = 3x_1x_2 + 6x_2 + 2x_1 + 4 \\
&6x_1 + 2x_2 = 6x_2 + 2x_1 \\
&4x_1 = 4x_2 \\
&x_1 = x_2
\end{aligned}$$

Thus, the given function is injective or one to one.

We know that, a function is surjective if for every value of $y \in Y$, there exist a $x \in X$ such that $f(x) = y$.

For that let’s take

$$y = \frac{3x+2}{x+2}$$

And try to find x such that f(x) = y for all y belongs to Y.

$$
\begin{aligned}
&y = \frac{3x+2}{x+2} \\
&y(x+2) = 3x+2 \\
&xy+2y = 3x+2 \\
&xy+2y-3x = 2 \\
&x(y-3) = 2-2y \\
&x = \frac{2-2y}{y-3}
\end{aligned}
$$

So, apparently, we see that for every y, there is an x as given above, but if you watch carefully, the value of x is undefined for y = 3. Saying so, in other words, we can very conveniently claim that for y = 3, there exist no x, Therefore, the given function is not surjective.

Solution 2:

Given that

$$f(x) = x^2 - 1 \text{ and } g(x) = 3x+5$$

Then

$$
\begin{aligned}
f \circ g(x) &= f(g(x)) \\
&= f(3x+5) \\
&= (3x+5)^2 - 1 \\
&= 9x^2 + 30x + 24
\end{aligned}
$$

$$
\begin{aligned}
g \circ f(x) &= g(f(x)) \\
&= g(x^2 - 1) \\
&= 3(x^2 - 1) + 5 \\
&= 3x^2 + 2
\end{aligned}
$$

Solution 3:

Given function is

$$
\begin{aligned}
&y = \frac{-2}{x-5} \\
&y(\mathrm{x}-5) = -2 \\
&xy - 5y = -2 \\
&xy = -2 + 5y \\
&x = \frac{5y-2}{y}
\end{aligned}
$$

Thus,

$$f^{-1}(\mathrm{y}) = \frac{5y-2}{y}$$

Solution 4:

Given that

$$y = g(x) \;\; = (2x - 1)$$
$$\mathrm{y}+1 = 2\,\mathrm{x}$$
$$\mathrm{x} = \frac{y+1}{2}$$

Which is the required inverse function.

Now, replacing x and y , we get

$$g^{-1}(\mathrm{x}) = \frac{x+1}{2}$$
$$g^{-1}(5) = \frac{5+1}{2} = 3$$

Solution 5:

Given that

$f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c and d are constants.

$$f \circ g(x) = f(g(x))$$
$$= f(cx+d)$$
$$= a(cx+d)+b$$
$$= acx + ad + b$$
$$g \circ f(x) = g(f(x))$$
$$= g(ax+b)$$
$$= c(ax+b)+d$$
$$= cax + cb + d$$
$$f \circ g(x) = g \circ f(x)$$
$$acx + ad + b = cax + cb + d$$

If

$$acx + ad + b = cax + cb + d$$
$$ad + b = cb + d$$
$$d(a-1) = b(c-1)$$
$$(a-1)/(c-1) = b/d$$