Solution to Practice questions Lecture 26-28

Solution 1:

We are to prove by contradiction that $4+3\sqrt{2}$ is an irrational number. So, let's assume that $4+3\sqrt{2}$ is a rational number. Then by definition of rational number, it can be written in the following form

$$4+3\sqrt{2}=\frac{a}{b} \quad \text{for some integers a and b with } b\neq 0$$

$$4+3\sqrt{2}=\frac{a}{b}$$
$$3\sqrt{2}=\frac{a}{b}-4$$
$$\sqrt{2}=\frac{a-4b}{3b}$$

Since a and b are integers, so are a-4b and 3b with $3b\neq 0$. This shows that $\sqrt{2}$ is a rational number. Which is a contradiction to the f act that $\sqrt{2}$ is an irrational number. This contradiction shows that our initial supposition was incorrect and thus, $4+3\sqrt{2}$ is not an irrational number that is $4+3\sqrt{2}$ is a rational number.

Solution 2:

We are to prove by contradiction that if $5n+1$ is odd then n is even. Suppose for some integer n , $5n+1$ is odd but n is not even. That is n is odd. So it can be written as

$$n=2m+1 \quad \text{for some integer } m.$$
$$5n+1=5(2m+1)+1$$
$$=10m+6$$
$$=2(5m+3)$$

This shows that $5n+1$ is equal to some multiple of 2 and so it is even which is a contradiction to our initial supposition. So, the correct supposition will be that if $5n+1$ is odd then n is even.

Solution 3:

We are to prove by contraposition that if n^2 is odd then n is odd. That is we are to prove that if n is even then n^2 is even.

Lets assume n is even. Then it can be written as a multiple of 2. That is

$$n = 2k \text{ for some integer } k$$
$$\begin{aligned} n^2 &= (2k)^2 \\ &= 4k^2 \\ &= 2(2k^2) \end{aligned}$$

Clearly, n^2 is a multiple of 2 so n^2 is also even.

This Proves that if n^2 is odd then n is odd.

Solution 4:

The following are pre and post conditions.

Pre-condition: The input variables *a* and *b* are positive integers.
Post-condition: The output variable *q* and *r* are positive integers such that

$a = b \cdot q + r$ and $0 \leq r < b$.

Solution 5:

We are to find the GCD (61114, 94).

Divide 61114 by 94

$$61114 = 94 \times 650 + 14$$

Divide 94 by 14

$$94 = 14 \times 6 + 10$$

Divide 14 by 10

$$14 = 10 \times 1 + 4$$

Divide 10 by 4

$$10 = 4 \times 2 + 2$$

Divide 4 by 2

$$4 = 2 \times 2 + 0$$

Thus, the GCD of 61114 and 94 is 2.