



Solution

$$\frac{\partial}{\partial x} (x^3 e^{-y} + y^3 \sec(\sqrt{x})) = 3e^{-y} x^2 + \frac{y^3 \sec(\sqrt{x}) \tan(\sqrt{x})}{2\sqrt{x}}$$

Steps

$$\frac{\partial}{\partial x} (x^3 e^{-y} + y^3 \sec(\sqrt{x}))$$

Treat y as a constant

Apply the Sum/Difference Rule: $(f \pm g)' = f' \pm g'$

$$= \frac{\partial}{\partial x} (x^3 e^{-y}) + \frac{\partial}{\partial x} (y^3 \sec(\sqrt{x}))$$

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$$\frac{\partial}{\partial x} (x^3 e^{-y}) = 3e^{-y} x^2$$

$$\frac{\partial}{\partial x} (x^3 e^{-y})$$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$$= e^{-y} \frac{\partial}{\partial x} (x^3)$$

Apply the Power Rule: $\frac{d}{dx} (x^a) = a \cdot x^{a-1}$

$$= e^{-y} \cdot 3x^{3-1}$$

Simplify

$$= 3e^{-y} x^2$$

Hide Steps

$$\frac{\partial}{\partial x} (y^3 \sec(\sqrt{x})) = \frac{y^3 \sec(\sqrt{x}) \tan(\sqrt{x})}{2\sqrt{x}}$$

$$\frac{\partial}{\partial x} (y^3 \sec(\sqrt{x}))$$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$$= y^3 \frac{\partial}{\partial x} (\sec(\sqrt{x}))$$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$$f = \sec(u), \quad u = \sqrt{x}$$

$$= y^3 \frac{\partial}{\partial u} (\sec(u)) \frac{\partial}{\partial x} (\sqrt{x})$$

$$\frac{\partial}{\partial u} (\sec(u)) = \sec(u) \tan(u)$$

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$$\frac{\partial}{\partial u} (\sec(u))$$

Apply the common derivative: $\frac{\partial}{\partial u} (\sec(u)) = \sec(u) \tan(u)$

$$= \sec(u) \tan(u)$$

$$\frac{\partial}{\partial x} (\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

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$$\frac{\partial}{\partial x} (\sqrt{x})$$

Apply radical rule: $\sqrt{a} = a^{\frac{1}{2}}$

$$= \frac{\partial}{\partial x} (x^{\frac{1}{2}})$$

Apply the Power Rule: $\frac{d}{dx} (x^a) = a \cdot x^{a-1}$

$$= \frac{1}{2} x^{\frac{1}{2}-1}$$

Simplify $\frac{1}{2} x^{\frac{1}{2}-1}$: $\frac{1}{2\sqrt{x}}$

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$$\frac{1}{2} x^{\frac{1}{2}-1}$$

$$x^{\frac{1}{2}-1} = x^{-\frac{1}{2}}$$

Hide Steps 

$$x^{\frac{1}{2}-1}$$

Hide Steps 

Join $\frac{1}{2} - 1$: $-\frac{1}{2}$

$$\frac{1}{2} - 1$$

Convert element to fraction: $1 = \frac{1 \cdot 2}{2}$

$$= -\frac{1 \cdot 2}{2} + \frac{1}{2}$$

Since the denominators are equal, combine the fractions: $\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$

$$= \frac{-1 \cdot 2 + 1}{2}$$

$$-1 \cdot 2 + 1 = -1$$

Hide Steps 

$$-1 \cdot 2 + 1$$

Multiply the numbers: $1 \cdot 2 = 2$

$$= -2 + 1$$

Add/Subtract the numbers: $-2 + 1 = -1$

$$= -1$$

$$= \frac{-1}{2}$$

Apply the fraction rule: $\frac{-a}{b} = -\frac{a}{b}$

$$= -\frac{1}{2}$$

$$= x^{-\frac{1}{2}}$$

$$= \frac{1}{2}x^{-\frac{1}{2}}$$

Apply exponent rule: $a^{-b} = \frac{1}{a^b}$

$$x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$\text{Multiply fractions: } \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

$$= \frac{1 \cdot 1}{2\sqrt{x}}$$

$$\text{Multiply the numbers: } 1 \cdot 1 = 1$$

$$= \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$= y^3 \sec(u) \tan(u) \frac{1}{2\sqrt{x}}$$

$$\text{Substitute back } u = \sqrt{x}$$

$$= y^3 \sec(\sqrt{x}) \tan(\sqrt{x}) \frac{1}{2\sqrt{x}}$$

$$\text{Simplify } y^3 \sec(\sqrt{x}) \tan(\sqrt{x}) \frac{1}{2\sqrt{x}} : \frac{y^3 \sec(\sqrt{x}) \tan(\sqrt{x})}{2\sqrt{x}}$$

Hide Steps 

$$y^3 \sec(\sqrt{x}) \tan(\sqrt{x}) \frac{1}{2\sqrt{x}}$$

$$\text{Multiply fractions: } a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$$

$$= \frac{1 \cdot y^3 \sec(\sqrt{x}) \tan(\sqrt{x})}{2\sqrt{x}}$$

$$\text{Multiply: } 1 \cdot y^3 = y^3$$

$$= \frac{y^3 \sec(\sqrt{x}) \tan(\sqrt{x})}{2\sqrt{x}}$$

$$= \frac{y^3 \sec(\sqrt{x}) \tan(\sqrt{x})}{2\sqrt{x}}$$

$$= 3e^{-y} x^2 + \frac{y^3 \sec(\sqrt{x}) \tan(\sqrt{x})}{2\sqrt{x}}$$