Lecture No -24 Sketching

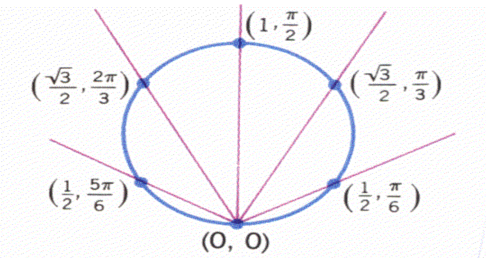
Draw graph of the curve having the equation r = sin θ

By substituting values for θ at increments of $\frac{\pi}{6}(30^0)$ and calculating r , we can construct The following table:

θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$
r = sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

θ (radians)	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r = sin θ	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	−1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0

Note that there are 13 pairs listed in Table, but only 6 points plotted in This is because the pairs from θ = π on yield duplicates of the preceding points. For example, (– ½, 7π/6) and (1/2, π/6) represent the same point. The points appear to lie on a circle.



hat this is indeed the case may be seen by expressing the given equation in terms of x and y. We first multiply the given equation through by r to obtain $r^2 = r\sin\theta$ which can be rewritten as

$$x^2 + y^2 = y \quad \text{or} \quad x^2 + y^2 - y = 0$$

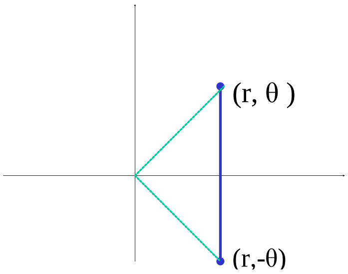
or on completing the square $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$ .This is a circle of radius 1\2 centered at the point (0,1/2) in the xy-plane.

Sketching of Curves in Polar Coordinates

1.SYMMETRY

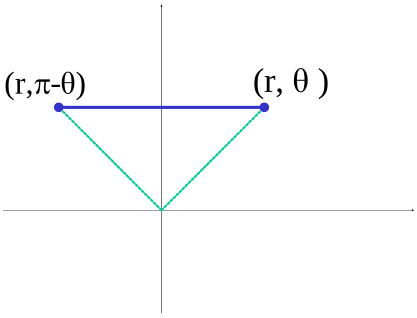
(i) Symmetry about the Initial Line

If the equation of a curve remains unchanged when (r, θ) is replaced by either $(r, -\theta)$ in its equation ,then the curve is symmetric about initial line.

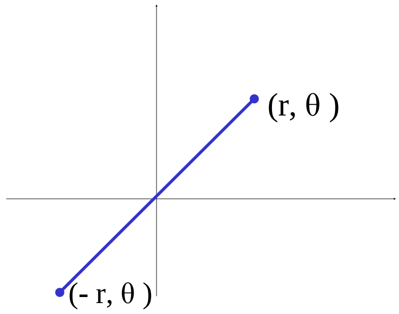


(ii) Symmetry about the y-axis

If when (r, θ) is replaced by either $(r, \pi - \theta)$ in The equation of a curve and an equivalent equation is obtained ,then the curve is symmetric about the line perpendicular to the initial i.e, the y-axis



(ii) Symmetry about the Pole
If the equation of a curve remains unchanged when either (-r, θ) or is substituted for (r, θ) in its equation ,then the curve is symmetric about the pole. In such a case ,the center of the curve.



2. Position Of The Pole Relative To The Curve

See whether the pole on the curve by putting r=0 in the equation of the curve and solving for θ.

3. Table Of Values

Construct a sufficiently complete table of values. This can be of great help in sketching the graph of a curve.

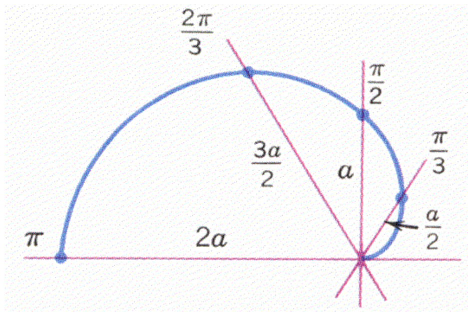
II Position Of The Pole Relative To The Curve.

When r = 0 , θ = 0 .Hence the curve passes through the pole.

III. Table of Values

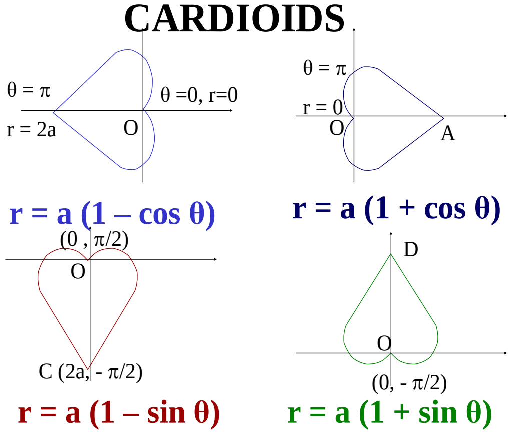
θ	0	π/3	π/2	2π/3	π
r=a (1–cosθ)	0	a/2	a	3a/2	2a

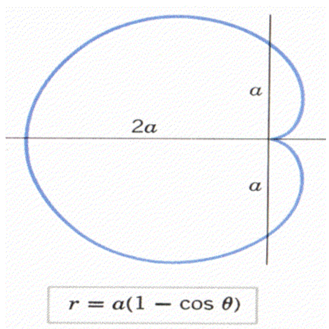
As θ varies from 0 to π, cos θ decreases steadily from 1 to – 1, and 1 – cos θ increases steadily from 0 to 2. Thus, as θ varies from 0 to π, the value of
r = a (1 – cos θ) will increase steadily from an initial value of r = 0 to a final value of
r = 2a.



On reflecting the curve in about the x-axis, we obtain the curve.

CARDIOIDS



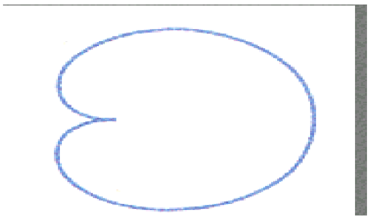


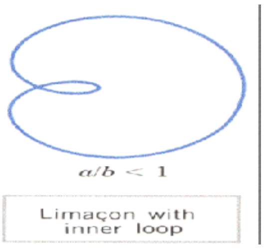
CARDIDOIDS AND LIMACONS

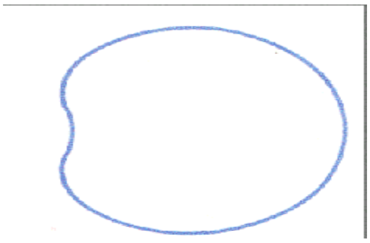
r=a+b sin θ, r = a–bsin θ

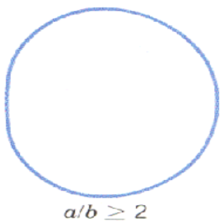
r=a+b cos θ, r = a–bcos θ

The equations of above form produce polar curves called limacons. Because of the heart-shaped appearance of the curve in the case a = b, limacons of this type are called cardioids. The position of the limacon relative to the polar axis depends on whether sin θ or cos θ appears in the equation and whether the + or – occurs.









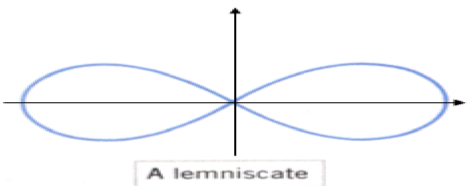
LEMINSCATE

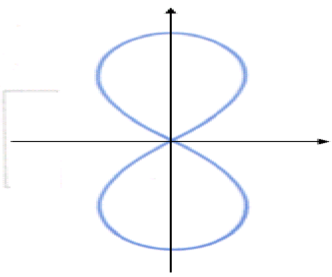
If a > 0, then equation of the form

$r^2 = a^2 \cos 2\theta$, $r^2 = -a^2 \cos 2\theta$

$r^2 = a^2 \sin 2\theta$, $r^2 = -a^2 \sin 2\theta$

represent propeller-shaped curves, called lemiscates (from the Greek word "lemnicos" for a looped ribbon resembling the Fig 8. The lemniscates are centered at the origin, but the position relative to the polar axis depends on the sign preceding the a^2 and whether sin 2θ or cos 2θ appears in the equation.

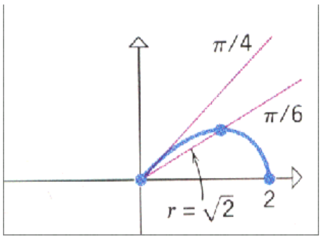


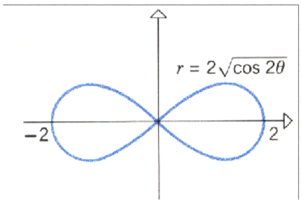


Example

$$r^2 = 4 \cos 2\theta$$

The equation represents a lemniscate. The graph is **symmetric** about the **x-axis** and the **y-axis**. Therefore, we can obtain each graph by first sketching the portion of the graph in the range $0 \le \theta < \pi/2$ and then reflecting that portion about the x- and y-axes.The curve passes through the origin when $\theta = \pi/4$, so the line $\theta = \pi/4$ is tangent to the curve at the origin.As θ varies from 0 to $\pi/4$, the value of $\cos 2\theta$ decreases steadily from 1 to 0, so that rdecreases steadily from 2 to 0.For θ in the range $\pi/4 < \theta < \pi/2$, the quantity $\cos 2\theta$ is negative, so there are no real values of r satisfying first equation. Thus, there are no points on the graph for such θ. The entire graph is obtained by reflecting the curve about the x-axis and then reflecting the resulting curve about the y-axis.



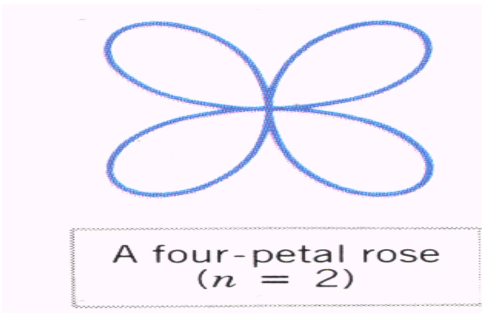


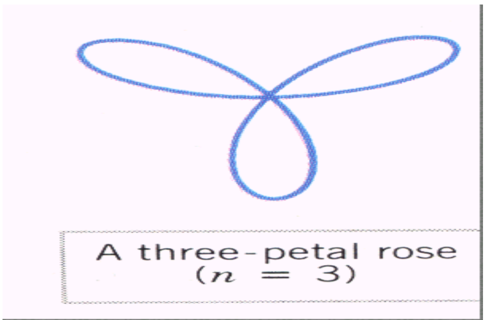
ROSE CURVES

Equations of the form

$r = a \sin n\theta$ and $r = a \cos n\theta$

represent flower-shaped curves called roses. The rose has **n equally spaced petals or loops if n is odd** and **2n equally spaced petals if n is even**





The orientation of the rose relative to the polar axis depends on the sign of the constant a and whether $\sin\theta$ or $\cos\theta$ appears in the equation.

SPIRAL

A curve that "winds around the origin" infinitely many times in such a way that r increases (or decreases) steadily as θ increases is called a spiral. The most common example is the spiral of Archimedes, which has an equation of the form.

$$r = a\theta \qquad (\theta \ge 0) \quad \text{or} \quad r = a\theta \qquad (\theta \le 0)$$

In these equations, θ is in radians and a is positive.

EXAMPLE

Sketch the curve **$r = \theta$ $(\theta \ge 0)$** in polar coordinates.

This is an equation of spiral with a = 1; thus, it represents an Archimedean spiral.

Since $r = 0$ when $\theta = 0$, the origin is on the curve and the polar axis is tangent to the spiral.

A reasonably accurate sketch may be obtained by plotting the intersections of the spiral with the x and y axes and noting that r increases steadily as θ increases. The intersections with the x-axis occur when

$\theta = 0, \ \pi, \ 2\pi, \ 3\pi, \ldots\ldots$
at which points r has the values
$r = 0, \ \pi, \ 2\pi, \ 3\pi, \ldots..$
and the intersections with the y-axis occur when
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots\ldots$
at which points r has the values
$r = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots\ldots$
Starting from the origin, the Archimedean spirals $r = \theta$ ($\theta \geq 0$) loops counterclockwise around the origin.