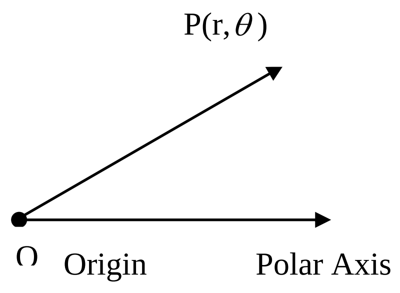
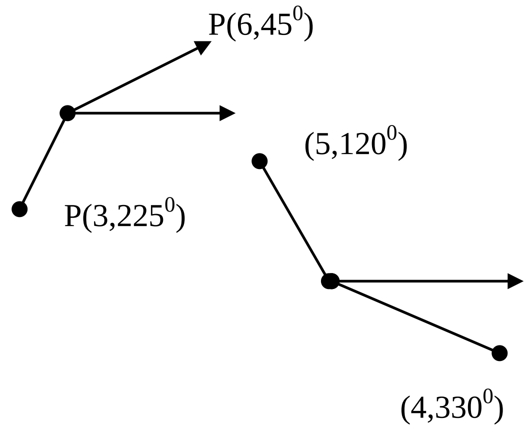
# Lecture No - 23 Polar Coordinate Systems

## POLAR COORDIANTE SYSTEMS

To form a polar coordinate system in a plane, we pick a fixed point O, called the origin or pole, and using the origin as an endpoint we construct a ray, called the polar axis. After selecting a unit of measurement, we may associate with any point P in the plane a pair of polar coordinates (r, θ), where r is the distance from P to the origin and θ measures the angle from the polar axis to the line segment OP.

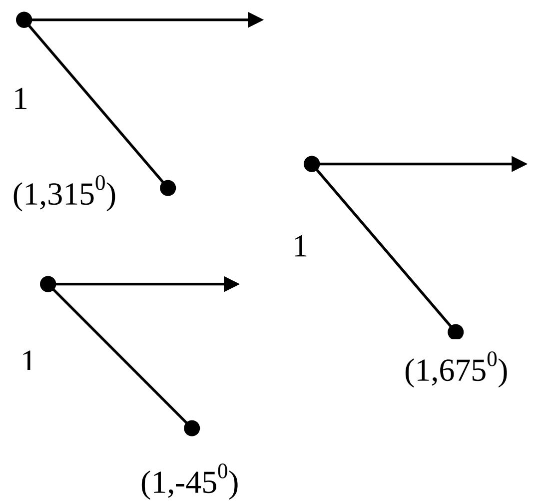



The number r is called the radial distance of P and θ is called a polar angle of P. In the points (6, 45°), (3, 225°), (5, 120°), and (4, 330°) are plotted in polar coordinate systems.




## THE POLAR COORDINATES OF A POINT ARE NOT UNIQUE.

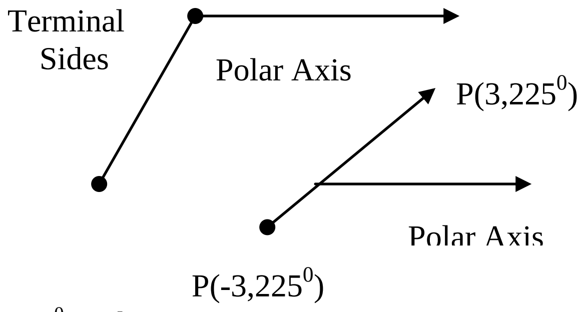
For example, the polar coordinates
(1, 315°), (1, – 45°), and (1, 675°)
all represent the same point
In general, if a point P has polar co-ordinate
(r, $\theta$), then for any integer n=0,1,2,3,…….
(r,$\theta$+n.$360^0$) and (r,$\theta$+n.$360^0$)
are also polar co-ordinates of p
In the case where P is the origin, the line line seg
Because there is no clearly defined polar angle ir
Polar angle θ may be used. Thus, for every θ ma
(0, θ ) is the origin.




## NEGATIVE VALUES OF R

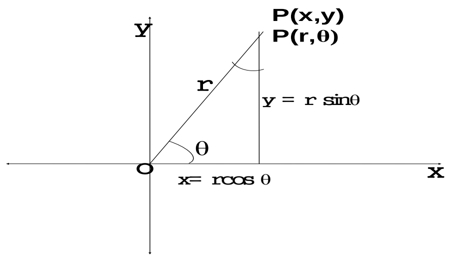
When we start graphing curves in polar coordinates, it will be desirable to allow negative values for r. This will require a special definition. For motivation, consider the point P with polar coordinates (3, $225^0$) We can reach this point by rotating the polar axis $225^0$ and then moving forward from the origin 3 units along the terminal side of the angle. On the other hand, we can also reach the point P by rotating the polar axis $45^0$ and then moving backward 3 units from the origin along the extension of the terminal side of the angle

This suggests that the point (3, 225 might also be denoted by ( − 3, 45 with the minus sign serving to indicate that the point is on the extension of the angle's terminal side rather than on the terminal side itself.

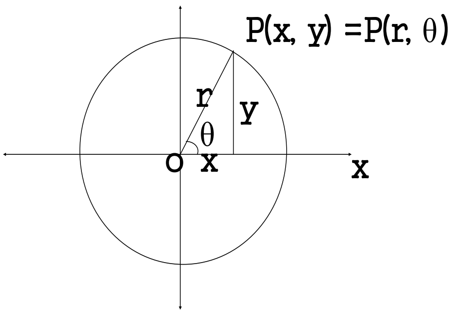



Since the terminal side of the angle $\theta + 180^0$ is the extension of the terminal side other angle $\theta$,We shall define.
(-r, $\theta$) and (r, $\theta + 180^0$ ) to be polar coordinates for the same point .
With r=3 and $\theta$ =$45^0$ in (2) if follows that (-3, $45^0$) and (-3, $225^0$) represent the same point.

## RELATION BETWEEN POLAR AND RECTANGULAR COORDINATES




## CONVERSION FORMULA FROM POLAR TO CARTESIAN COORDINATES AND VICE VERSA

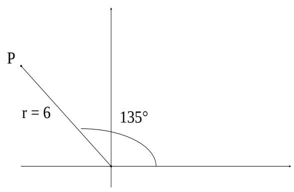



$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$x^2 + y^2 = r^2$$
$$y/x = \tan\theta$$

**Example**
Find the rectangular coordinates of the point P whose polar co-ordinates are $(6,135^0)$
**Solution:**
Substituting the polar coordinates
r = 6 and $\theta$=$135^0$ in x = cos$\theta$ and y=sin$\theta$ yeilds

$$x = 6\cos 135° = 6(-\sqrt{2}/2) = -3\sqrt{2}$$
$$y = 6\sin 135° = 6(\sqrt{2}/2) = 3\sqrt{2}$$




Thus, the rectangular coordinates of the point P are $(-3\sqrt{2}, 3\sqrt{2})$

**Example:**

Find polar coordinates of the point P whose rectangular coordinates are $(-2, 2\sqrt{3})$
**Solution**:

We will find polar coordinates (r, $\theta$) of P such that r >0 and $0 \le \theta \le 2\pi$ .

$$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$$

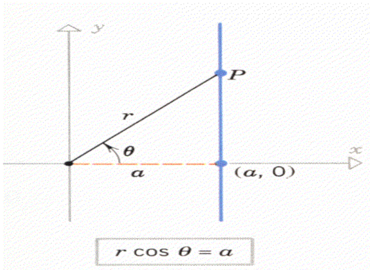
$$\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{-2} = -\sqrt{3} \Rightarrow \theta = \tan^{-1}(-\sqrt{3}) = \frac{2\pi}{3}$$

From this we have (-2,2 $\sqrt{3}$ ) lies in the second quadrant of P. All other Polar co-ordinates of P have the form

$$(4, \frac{2\pi}{3} + 2n\pi) \quad or \quad (-4, \frac{5\pi}{3} + 2n\pi) \quad , \quad Where\ n\ is\ \text{int}eger$$

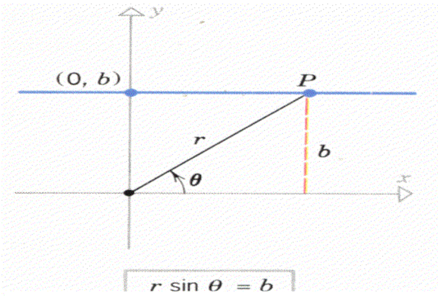
**LINES IN POLAR COORDIANTES**

A line perpendicular to the x-axis and passing through the point with xy co-ordinates with (a,0) has the equation x=a . To express this equation in polar co-ordinates we substitute x = r cos $\theta$ $\Rightarrow$ a = rcos $\theta$ --------(1)




This result makes sense geometrically since each point P (r, θ) on this line will yield the value a for r cos θ.

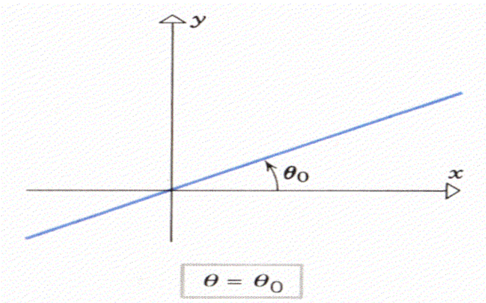
A line parallel to the x-axis that meets the y-axis it the point with xy-coordinates (0, b) has the equation y = b.




Substituting y = r sin θ yields.

r sin θ = b (2)

as the polar equation of this line. This makes sense geometrically since each point P (r, θ) on this line will yield the value b for r sin θ




For Any constant $\theta_0$, the equation

$\theta = \theta_0$ (3)

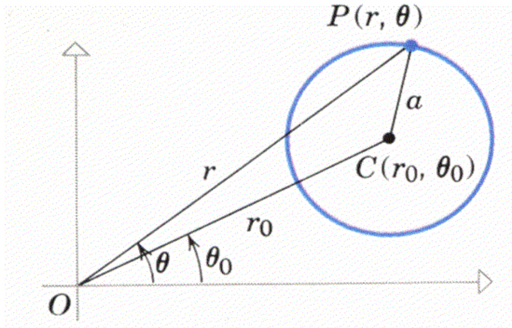
is satisfied by the coordinates of all points of the form P (r, $\theta_0$), regardless of the value of r. Thus, the equation represents the line through the origin making an angle of $\theta_0$ (radians) with the polar axis.

By substitution x = rcosθ and y = rsinθ in the equation Ax +By +C = 0 . We obtain the general polar form of the line,

$$r\,(A\cos\theta + B\sin\theta) + C = 0$$

## CIRCLES IN POLAR COORDINATES

Let us try to find the polar equation of a circle whose radius is a and whose center has polar coordinates ($r_0$, $\theta_0$). If we let P(r, θ) be an arbitrary point on the circle, and if we apply the law of cosines to the triangle OCP we obtain

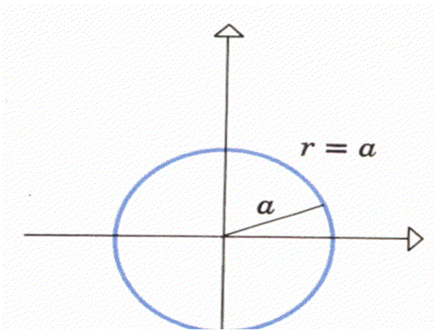



$r^2 - 2rr_0 \cos(\theta - \theta_0) + r_0^2 = a^2$ (1)

## SOME SPECIAL CASES OF EQUATION OF CIRCLE IN POLAR COORDINATES

A circle of radius a, centered at the origin, has an especially simple polar equation. If we let $r_0 = 0$ in (1), we obtain $r^2 = a^2$ or, since $a \geq 0$, r = a

This equation makes sense geometrically since the circle of radius a, centered at the origin, consists of all points P (r, θ) for which r = a, regardless of the value of θ




If a circle of radius a has its center on the x-axis and passes through the origin, then the polar coordinates of the center are either

(a, 0) or (a, π)

depending on whether the center is to the right or left of the origin