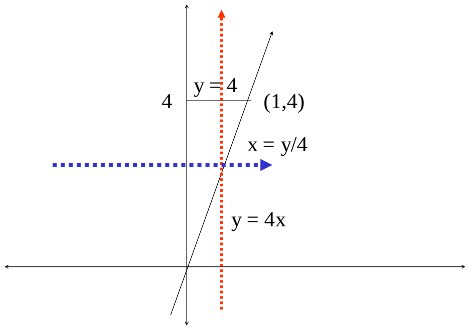
Lecture No -21 Examples

$$\int_0^1 \int_{4x}^4 e^{-y^2}\, dy\, dx$$



Reversing the order of integration

$$\int_0^4 \int_0^{y/4} e^{-y^2}\, dx\, dy$$

$$= \int_0^4 \left| x\, e^{-y^2} \right|^{y/4} dy \quad = \int_0^4 \frac{y}{4}\, e^{-y^2}\, dy$$

$$= \frac{-1}{8} \int_0^4 e^{-y^2} (-2y)\, dy$$

$$= -\frac{1}{8} \left| e^{-y^2} \right|_0^4 = -\frac{1}{8} \left| e^{-16} - c^0 \right|$$

$$= \frac{1}{8}\left(1 - \frac{1}{e^{16}}\right)$$

Example

Calculate

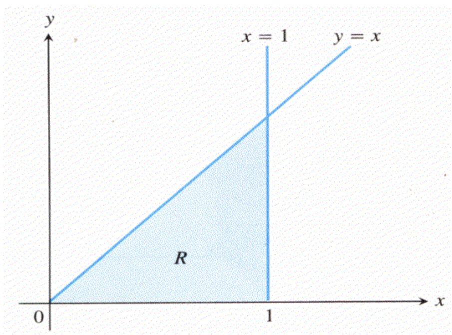
$$\iint_R \frac{\sin x}{x}\, dA.$$

where R is the triangle in the xy- plane bounded by the x-axis ,the line y=x and the line x=1

We integrate first with respect to y and then with respect to x, we find

$$\int_0^1 \left(\int_0^x \frac{\sin x}{x}\, dy \right) dx$$

$$= \int_0^1 \left(y \frac{\sin x}{x} \Big]_{y=0}^{y=x} \right) dx$$



$$= \int_0^1 \sin x \, dx = -\cos(0) + 1 \approx 0.46$$

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EXAMPLE

$$\int_0^2 \int_{y/2}^1 \mathbf{e}^{\mathbf{x^2}} \, dxdy$$

Since there is no elementary antiderivative of e^{x^2}, the integral

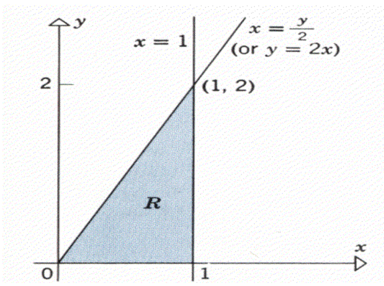
$$\int_0^2 \int_{y/2}^1 e^{x^2} \, dxdy$$

cannot be evaluated by performing the x-integration first.

To evaualte this integral , we express is as an equivalent iterated integral with the order if integration reversed . For the inside integration, y is fixed and x varies from he line x = y/2 to the line x = 1. For the outside integration, y varies from 0 to 2, so the given iterated integral is equal to a double integral over the triangular region R.

To reverse the order of integration, we treat R as a type I region, which enables us to write the given integral as

$$\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} dxdy$$



By changing the order of integration we get,

$$\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} dxdy = \int_0^1 \int_0^{2x} e^{x^2} dydx$$

$$= \int_0^1 [e^{x^2} y]_{y=0}^{2x} \, dx$$

$$= \int_0^1 2xe^{x^2} \, dx$$

$$= e^{x^2} \Big|_0^1 = e - 1$$

EXAMPLE

Use a double integral to find the volume of the solid that is bounded above by the palne Z=4-x-y and below by the rectangle $R = \{(x,y): 0 \le x \le 1, 0 \le y \le 2\}$

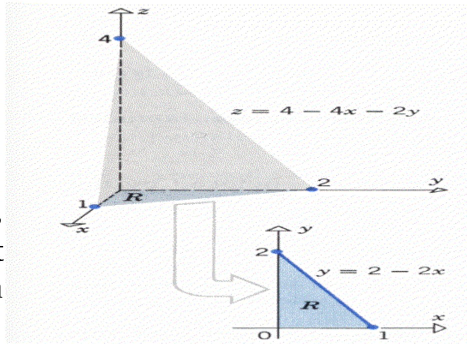
$$V=\iint_R (4-x-y)\, dA$$
$$=\int_0^2\int_0^1 (4-x-y)\, dx\, dy$$
$$=\int_0^2 \left[4x-\frac{x^2}{2}-xy\right]_{x=0}^{1} dy$$
$$=\int_0^2\left(\frac{7}{2}-y\right) dy = \left[\frac{7}{2}y-\frac{y^2}{2}\right]_0^2 = 5$$

EXAMPLE

Use a double integral to find the volume of the tetrahedron bounded by the coordinate planes and the plane z=4-4x-2y The tetrahedron is bounded above by the plane.

z=4-4x-2y ------------------(1)

and below by the triangular region R



Thus, the volume is given by

$$V = \iint_R (4-4x-2y)\, dA$$

The region R is bounded by the x-axis, the y-axis, and the line y = 2 – 2x [set z = 0 in (1)], so that treating R as a type I region yields.

$$V = \iint_R (4-4x-2y)\, dA$$
$$= \int_0^1\int_0^{2-2x} (4-4x-2y)\, dy\, dx$$
$$= \int_0^1 [4y-4xy-y^2]_{y=0}^{2-2x}\, dx$$
$$= \int_0^1 (4-8x+4x^2)dx$$
$$= \frac{4}{3}$$

Find the volume of the solid bounded by the cylinder x2+y2 = 4 and the planes y + z = 4 and z = 0.

The solid is bounded above by the plane z = 4 – y and below by the region R within the circle $x^2 + y^2 = 4$.

The volume is given by

$$V = \iint_R (4-y)\, dA$$

Treating R as a type I region we obtain

$$V = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4-y)\, dy\, dx$$
$$= \int_{-2}^{2}\left[4y-\frac{1}{2}y^2\right]_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx$$
$$= \int_{-2}^{2} 8\sqrt{4-x^2}\, dx$$

$$= 8\left| \frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2} \right|_{-2}^{2}$$

$$= 8\left| 2\sin^{-1}(1) - 2\sin^{-1}(-1) \right|$$

$$= 8[2(\frac{\pi}{2}) + 2(\frac{\pi}{2})]$$

$$= 8(2\pi) = 16\pi$$

EXAMPLE

Use double integral to find the volume of the solid that is bounded above by the paraboiled $z=9x^2 + y^2$,below by the plane $z=0$ and laterally by the planes

$x = 0$, $y = 0$, $x = 3$, $y = 2$

$$\text{Volume} = \int_0^3\int_0^2 (9x^2 + y^2)\,dy\,dx$$

$$= \int_0^3 \left[9x^2y + \frac{y^3}{3}\right]_0^2 dx$$

$$= \int_0^3 \left(18x^2 + \frac{8}{3}\right) dx$$

$$= \left| 6x^3 + \frac{8}{3}x \right|_0^3$$

$$= 6(27) + 8$$

$$= 170$$