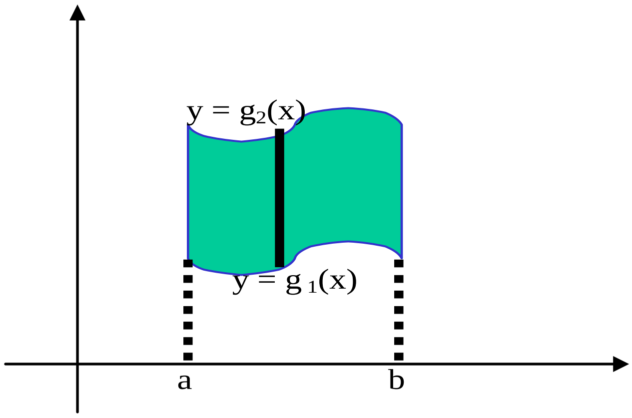
Lecture No -20 Double integral for non-rectangular region

Double integral for non-rectangular region

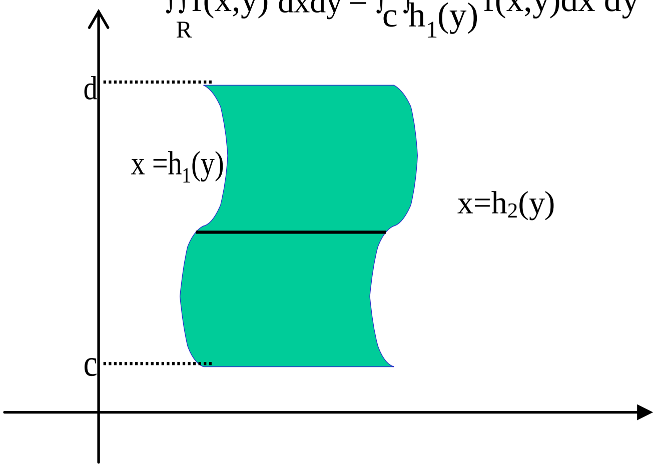
Type I region is bounded the left and right by vertical lines x=a and x=b and is bounded below and above by curves $y=g_1(x)$ and $y=g_2(x)$ where $g_1(x) \le g_2(x)$ for $a\le x\le b$

$$\iint_R f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$$



Type II region is bounded below and above by the horizontal lines y=c and y=d and is bounded on the left and right by the continuous curves $x= h_1(y)$ and $x=h_2(y)$ satisfying $h_1(y) \le h_2(y)$ for $c\le y\le d$

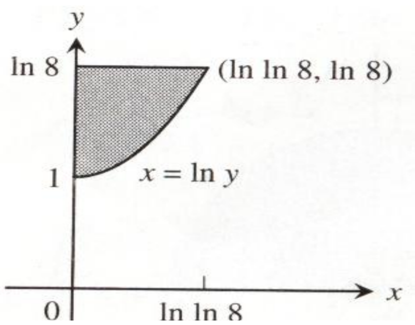
$$\iint_R f(x,y)\, dxdy = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dx\, dy$$



Write double integral of the function *f(x,y)* on the region whose sketch is given

$$\int_1^{\ln 8} \int_0^{\ln y} f(x,y)\, dx$$

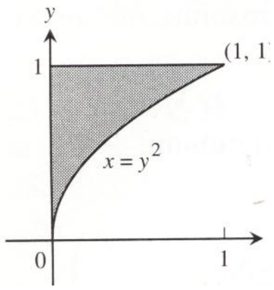
$$\int_0^{\ln (\ln 8)} \int_{e^x}^{\ln 8} f(x,y) \quad y$$



Write double integral of the function *f(x,y)* on the region whose sketch is given

$$\int_0^1\int_0^{y^2} f(x,y)\,dx\,dy$$

$$\int_0^1\int_{\sqrt{x}}^1 f(x,y)\,dy\,dx$$



EXAMPLE

Draw the region and evaluate an equivalent integral with the order of integration reversed

$$\int_0^2\int_{x^2}^{2x} (4x+2)\,dy\,dx$$

The region of integration is given by the inequalities $x^2 \leq y \leq 2x$ and $0 \leq x \leq 2$.

$$\int_0^4\int_{y/2}^{\sqrt{y}} (4x+2)\,dx\,dy.$$

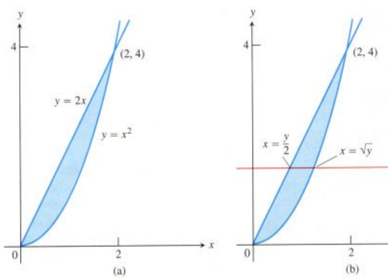
$$= \int_0^4 \left|2x^2 + 2x\right|_{y/2}^{\sqrt{y}} dy$$

$$= \int_0^4 \left[2y + 2\sqrt{y} - \frac{y}{2} - y\right] dy$$

$$= \left|y^2 + \frac{4}{3}y^{3/2} - \frac{y^3}{6} - \frac{y^2}{2}\right|_0^4$$

$$= \left[16 + \frac{4}{3}(4)^{3/2} - \frac{(4)^3}{6} - 8\right]$$

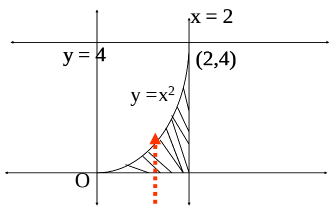
$$= 16 + \frac{4}{3}(8) - \frac{64}{6} - 8 = 8$$

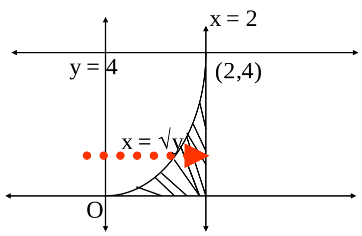


EXAMPLE

Evaluate $I = \int_0^4\int_{\sqrt{y}}^2 y\cos x^5\,dx\,dy$ The integral is over the region $0 \leq y \leq 4$, $x = \sqrt{y}$ and $x = 2$

$$I = \int_0^2\int_0^{x^2} y\cos x^5\,dy\,dx$$





$$= \int_0^2 \left[\frac{y^2}{2}\cos x^5\right]_0^{x^2} dx$$

$$= \int_0^2 \frac{x^4}{2}\cos x^5\, dx$$

$$= \frac{1}{10}\int_0^2 \cos x^5 \,.\, (5x^4)\, dx$$

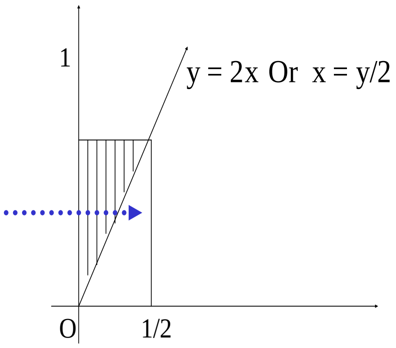
$$= \left[\frac{1}{10}\sin x^5\right]_0^2 = \frac{1}{10}\sin 32$$

Evaluate $I = \int_0^{1/2}\int_{2x}^{1} e^{y^2}\, dy\, dx$

The integral cannot be evaluated I the given order since e^{y^2} has no antiderivative.We shall change the order of integration. The region R which integration is performed is given by $0 \le x \le \frac{1}{2}$, $y = 2x$ and $y = 1$

This region is also enclosed by

$x = 0$, $x = \frac{y}{2}$ and $0 \le y \le 1$



Thus

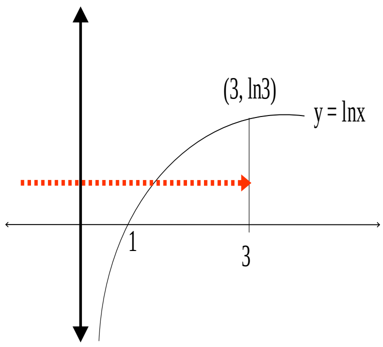
$$I = \int_0^1\int_0^{y/2} e^{y^2}\, dx\, dy$$

$$= \int_0^1 \frac{y}{2} e^{y^2}\, dy$$

$$= \left[\frac{1}{4} e^{y^2}\right]_0^1 = \frac{1}{4}(e-1)$$

$$\int_1^3\int_0^{\ln x} x\, dy\, dx$$

Reversing the order of integration



$$= \int_0^{\ln 3}\int_{e^y}^{3} x\, dx\, dy$$

$$= \int_0^{\ln 3} \left|\frac{x^2}{2}\right|_{e^y}^{3} dy = \frac{1}{2}\int_0^{\ln 3} [9 - e^{2y}]\, dy$$

$$= \frac{1}{2}\left|9y - \frac{e^{2y}}{2}\right|_{0}^{\ln 3}$$

$$= \frac{1}{2}\left[9\ln 3 - \frac{e^{2\ln 3}}{2} + \frac{e^{0}}{2}\right]$$

$$= \frac{1}{2}\left[9\ln 3 - \frac{9}{2} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[9\ln 3 - 4\right]$$

$$= \frac{9}{2}\ln 3 - 2$$

<u>Over view of Lecture # 20</u>

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