Lecture No-3 Elements of three dimensional geometry

Distance formula in three dimension

Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points such that PQ is not parallel to one of the coordinate axis Then $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ Which is known as Distance fromula between the points P and Q.

Example of distance formula

Let us considerthe points A (3, 2, 4), B (6, 10, –1),
and C (9, 4, 1)
Then

$$|AB| = \sqrt{(6-3)^2 + (10-2)^2 + (-1-4)^2} = \sqrt{98} = 7\sqrt{2}$$
$$|AC| = \sqrt{(9-3)^2 + (4-2)^2 + (1-4)^2} = \sqrt{49} = 7$$
$$|BC| = \sqrt{(9-6)^2 + (4-10)^2 + (1+1)^2} = \sqrt{49} = 7$$

Mid point of two points

If R is the middle point of the line segment PQ, then the co-ordinates of the middle points are
x= (x1+x2)/2 ,
y= (y1+y2)/2 ,
z= (z1+z2)/2

Let us consider tow points A(3,2,4) and B(6,10,-1)
Then the co-ordinates of mid point of AB is

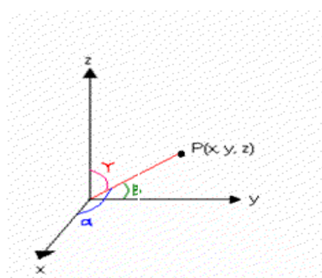
[(3+6)/2,(2+10)/2,(4-1)/2]
= (9/2,6,3/2)

Direction Angles

The **direction angles** α β, γ of a lineare defined as

α = Angle between lineand the positive x-axis
β = Angle between line and the positive y-axis
γ = Angle between lineand the positive z-axis.

By definition, each of these angles lies between 0 and π

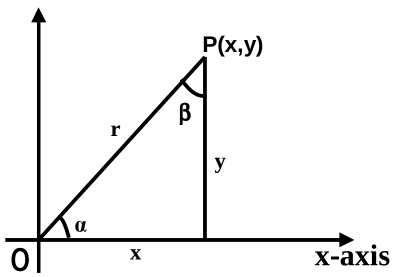


Direction Ratios

Cosines of direction angles are called direction cosines
Any multiple of direction cosines are called direction numbers or direction ratios of the line L.

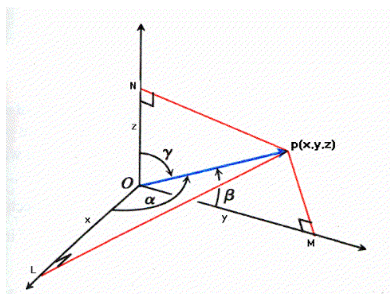
Given a point, finding its Direction cosines

y-axis



From triangle we can write
$\cos \alpha = x/r$
$\cos \beta = y/r$

Direction angles of a Line



The angles which a line makes with positive x,y and z-axis are known as Direction Angles. In the above figure the blue line has direction angles as α,□□□and□□□which are the angles which blue line makes with x,y and z-axis respectively.

Direction cosines:

Now if we take the cosine of the Direction Angles of a line then we get the Direction cosines of that line. So the Direction Cosines of the above line are given by

$$\cos \alpha = \frac{x}{OP} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\cos \beta = \frac{y}{OP} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

Similarly,

$$\cos \gamma = \frac{z}{OP} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1.$$

Direction cosines and direction ratios of a line joining two points

•For a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, x_2)$ the direction ratios are

$x_2 - x_1, y_2 - y_1, z_2 - z_1$ and the directions cosines are $\frac{x_2 - x_1}{|PQ|}, \frac{y_2 - y_1}{|PQ|}$ *and* $\frac{z_2 - z_1}{|PQ|}$.

Example For a line joining two points P(1,3,2) and Q(7,-2,3) the direction ratios are

7 - 1, -2 – 3 , 3 – 2

6 , -5 , 1

and the directions cosines are

6/√62 , -5/√62 , 1/ √62

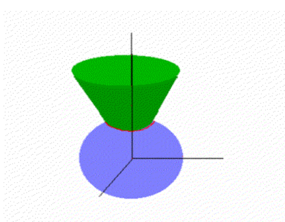
In two dimensional space the graph of an equation relating the variables x and y is the set of all point (x, y) whose co-ordinates satisfy the equation. Usually, such graphs are curves. In three dimensional space the graph of an equation relating the variables x, y and z is the set of all point (x, y, z) whose co-ordinates satisfy the equation. Usually, such graphs are surfaces.

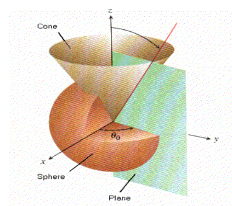
Intersection of two surfaces

•Intersection of two surfaces is a curve in three dimensional space.

•It is the reason that a curve in three dimensional space is represented by two equations representing the intersecting surfaces.

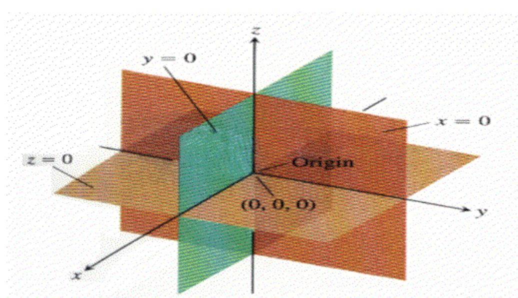
Intersection of Cone and Sphere





Intersection of Two Planes

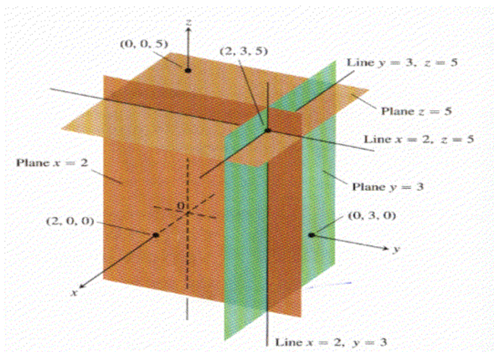
If the two planes are not parallel, then they intersect and their intersection is a straight line. Thus, two non-parallel planes represent a straight line given by two simultaneous linear equations in x, y and z and are known as non-symmetric form of equations of a straight line.



The planes $x = 0$, $y = 0$, and $z = 0$ divide space into eight octants.

REGION	DESCRIPTION	EQUATION
xy-plane	Consists of all points of the form (x, y, 0)	z = 0
xz-plane	Consists of all points of the form (x, 0, z)	y = 0
yz-plane	Consists of all points of the form (0, y, z)	x = 0
x-axis	Consists of all points of the form (x, 0, 0)	y = 0, z = 0
y-axis	Consists of all points of the form (0, y, 0)	z = 0, x = 0
z-axis	Consists of all points of the form (0, 0, z)	x = 0, y = 0

Planes parallel to Co-ordinate Planes



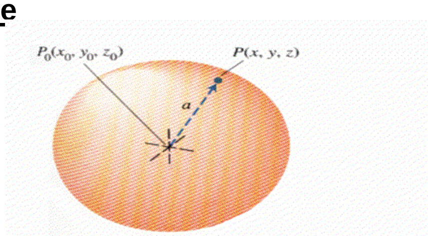
General Equation of Plane

Any equation of the form

$$ax + by + cz + d = 0$$

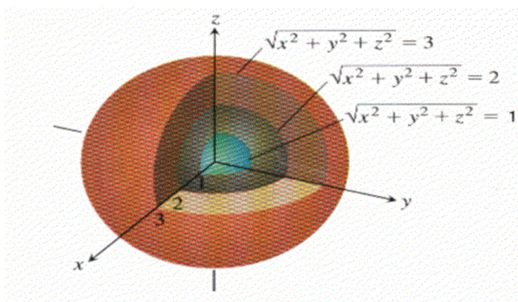
where a, b, c, d are real numbers,represent a plane.

Sphere



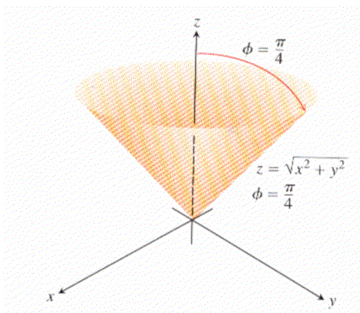
The standard equation of the sphere of radius a centered at (x_0, y_0, z_0) is

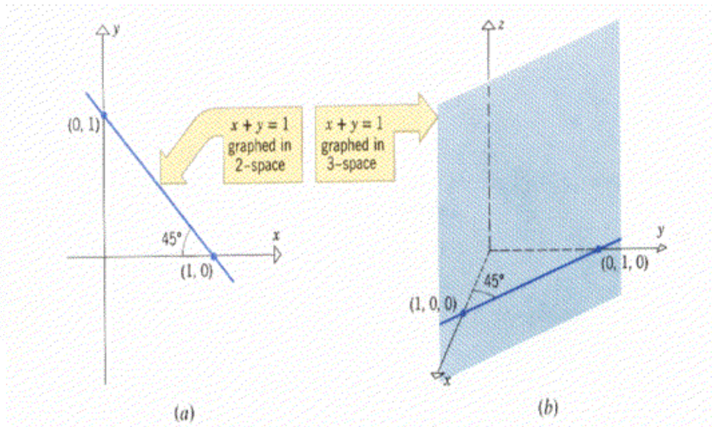
$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$.

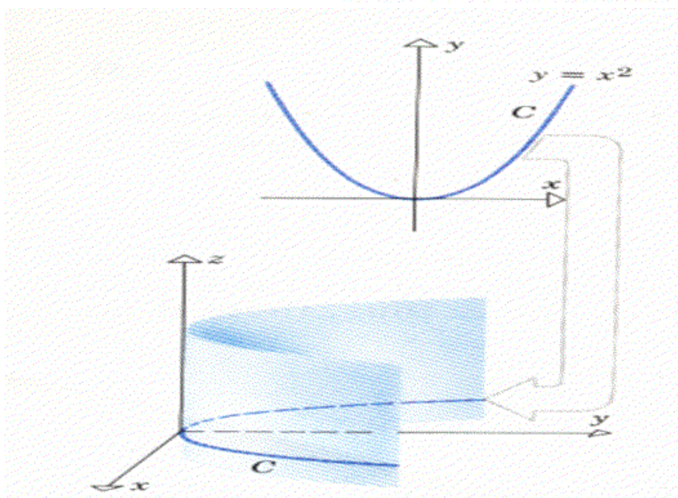


The level surfaces of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ are concentric spheres.

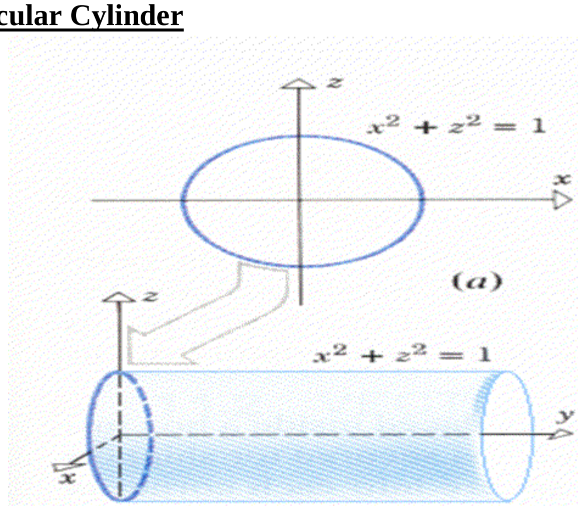
Right Circular Cone



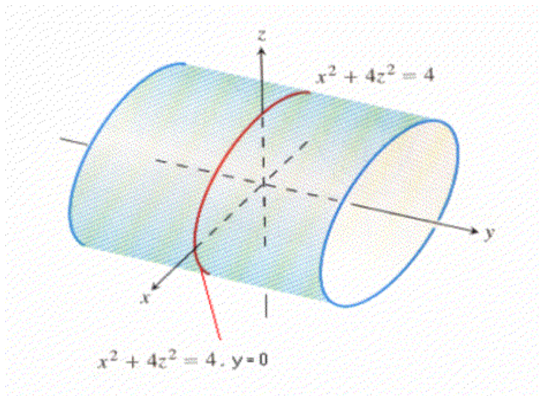


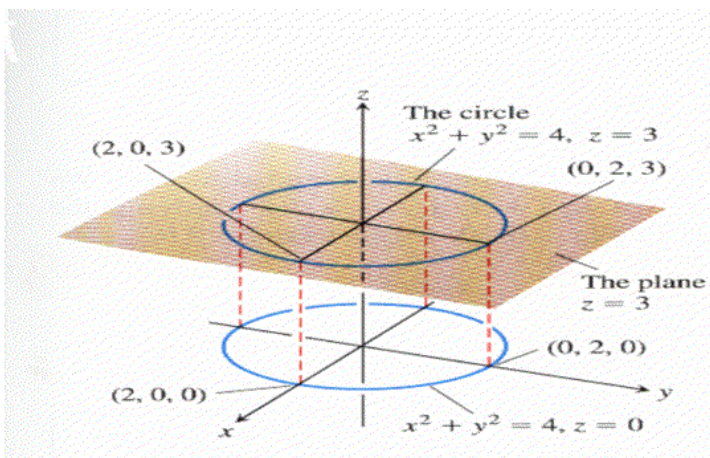


Horizontal Circular Cylinder



Horizontal Elliptic Cylinder





The circle $x^2 + y^2 = 4$, $z = 3$.

Overview of Lecture # 3

Chapter # 14

Three Diamentional Space

Page # 657

Book **CALCULUS by HOWARD ANTON**