Lecture No-2 Values of functions:

Consider the function $f(x) = 2x^2 - 1$, then $f(1) = 2(1)^2 - 1 = 1$, $f(4) = 2(4)^2 - 1 = 31$, $f(-2) = 2(-2)^2 - 1 = 7$

$f(t-4) = 2(t-4)^2 - 1 = 2t^2 - 16t + 31$

These are the values of the function at some points.

Example

Now we will consider a function of two variables, so consider the function $f(x,y) = x^2y+1$ then $f(2,1) = (2^2)1+1=5$, $f(1,2) = (1^2)2+1=3$, $f(0,0) = (0^2)0+1=1$, $f(1,-3) = (1^2)(-3)+1=-2$, $f(3a,a) = (3a)^2a+1=9a^3+1$, $f(ab,a-b) = (ab)^2(a-b)+1=a^3b^2-a^2b^3+1$

These are values of the function at some points.

Example:

Now consider the function $f(x,y) = x + \sqrt[3]{xy}$ then

(a) $f(2,4) = 2 + \sqrt[3]{(2)(4)} = 2 + \sqrt[3]{8} = 2 + 2 = 4$

(b) $f(t,t^2) = t + \sqrt[3]{(t)(t^2)} = t + \sqrt[3]{t^3} = t + t = 2t$

(c) $f(x,x^2) = x + \sqrt[3]{(x)(x^2)} = x + \sqrt[3]{x^3} = x + x = 2x$

(d) $f(2y^2,4y) = 2y^2 + \sqrt[3]{(2y^2)(4y)} = 2y^2 + \sqrt[3]{8y^3} = 2y^2 + 2y$

Example:

Now again we take another function of three variables

$$f(x,y,z) = \sqrt{1 - x^2 - y^2 - z^2} \text{ Then}$$

$$f(0,\frac{1}{2},\frac{1}{2}) = \sqrt{1 - 0 - (\frac{1}{2})^2 - (\frac{1}{2})^2} = \sqrt{\frac{1}{2}}$$

Example:

Consider the function $f(x,y,z) = xy^2z^3+3$ then at certain points we have

$f(2,1,2) = (2)(1)^2(2)^3+3=19$, $f(0,0,0) = (0)(0)^2(0)^3+3=3$, $f(a,a,a) = (a)(a)^2(a)^3+3=a^6+3$
$f(t,t^2,-t) = (t)(t^2)^2(-t)^3+3=-t^8+3$, $f(-3,1,1) = (-3)(1)^2(1)^3+3=0$

Example:

Consider the function $f(x,y,z) = x^2y^2z^4$ where $x(t) = t^3$ $y(t) = t^2$ and $z(t) = t$

(a) $f(x(t),y(t),z(t)) = [x(t)]^2[y(t)]^2[z(t)]^4 = [t^3]^2[t^2]^2[t]^4 = t^{14}$
(b) $f(x(0),y(0),z(0)) = [x(0)]^2[y(0)]^2[z(0)]^4 = [0^3]^2[0^2]^2[0]^4 = 0$

Example:

Let us consider the function $f(x,y,z) = xyz + x$ then

$$f(xy,y/x,xz) = (xy)(y/x)(xz) + xy = xy^2z+xy.$$

Example:

Let us consider $g(x,y,z) = z\,\mathrm{Sin}(xy)$, $u(x,y,z) = x^2z^3$, $v(x,y,z) = Pxyz$,

$w(x,y,z) = \dfrac{xy}{z}$ Then.

$g(u(x,y,z), v(x,y,z), w(x,y,z)) = w(x,y,z)\,\mathrm{Sin}(u(x,y,z)\,v(x,y,z))$

Now by putting the values of these functions from the above equations we get

$$g(u(x,y,z), v(x,y,z), w(x,y,z)) = \frac{xy}{z}\ \text{Sin}[(x^2z^3)(\ Pxyz)] = \frac{xy}{z}\ \text{Sin}[(Pyx^3z^4)].$$

Example:

Consider the function $g(x,y) = y\ \text{Sin}(x^2y)$ and $u(x,y) = x^2y^3$ $v(x,y) = \pi xy$ Then

$$g(u(x,y), v(x,y)) = v(x,y)\ \text{Sin}([u(x,y)\ ]^2\ v(x,y))$$

By putting the values of these functions we get

$$g(u(x,y), v(x,y)) = \pi xy\ \text{Sin}([x^2y^3]^2\ \pi xy) = \pi xy\ \text{Sin}(x^5y^7\ ).$$

Function of One Variable

A function f of one real variable x is a rule that assigns a unique real number f(x) to each point x in some set D of the real line.

Function of two Variables

A function f in two real variables x and y, is a rule that assigns unique real number f (x,y) to each point (x,y) in some set D of the xy-plane.

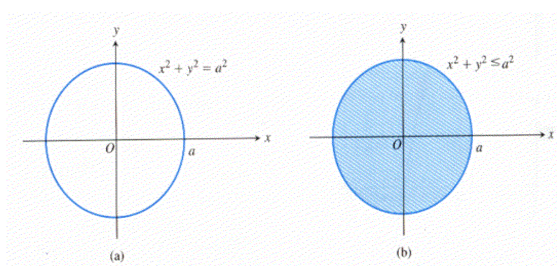
Function of three variables:

A function f in three real variables x, y and z, is a rule that assigns a unique real number f (x,y,z) to each point (x,y,z) in some set D of three dimensional space.

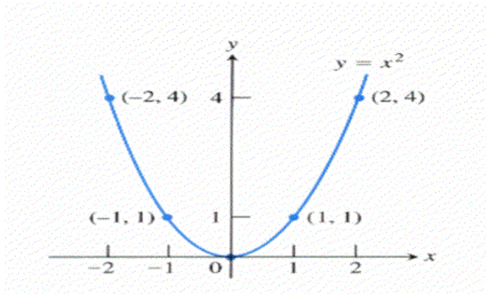
Function of n variables:

A function f in n variable real variables $x_1, x_2, x_3, \ldots\ldots, x_n$, is a rule that assigns a unique real number $w = f(x_1, x_2, x_3, \ldots\ldots, x_n)$ to each point $(x_1, x_2, x_3, \ldots\ldots, x_n)$ I n some set D of n dimensional space.

Circles and Disks:

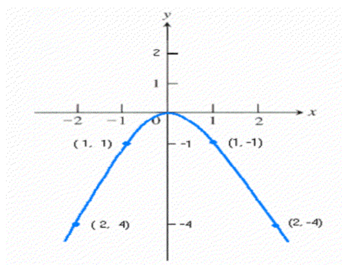


PARABOLA

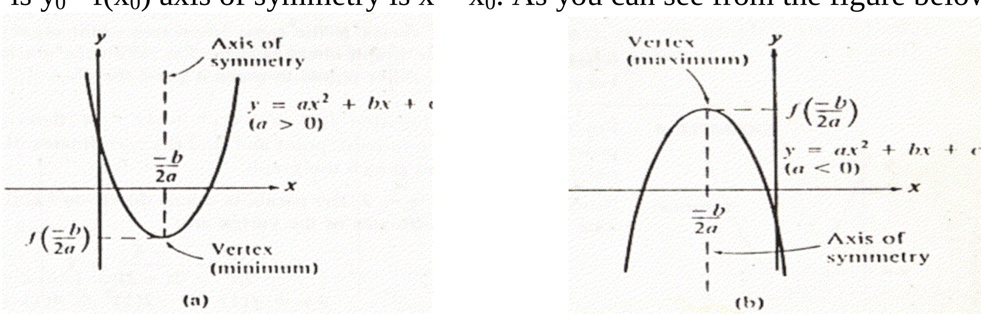


The parabola $y = x^2$.

Parabola $y = -x^2$



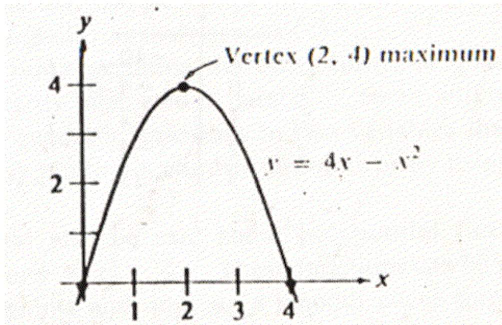
General equation of the Parabola opening upward or downward is of the form
$y = f(x) = ax^2+bx + c$. Opening upward if $a > 0$. Opening downward if $a < 0$.
x co-ordinate of the vertex is given by $x_0 = -b/2a$. So the y co-ordinate of the vertex is $y_0= f(x_0)$ axis of symmetry is $x = x_0$. As you can see from the figure below



<u>Sketching of the graph of parabola $y = ax^2+bx + c$</u>
Finding vertex: x – co-ordinate of the vertex is given by $x_0= - b/2a$
So, y – co-ordinate of the vertex is $y_0= a\,x_0^2+b\,x_0 + c$. Hence vertex is $V(x_0 , y_0)$.

<u>Example:</u> Sketch the parabola $y = - x^2 + 4x$
Solution: Since $a = -1 < 0$, parabola is opening downward. Vertex occurs at $x = - b/2a = (-4)/2(-1) =2$. Axis of symmetry is the vertical line $x = 2$. The y-co-ordinate of the vertex is$y = -(2)^2 + 4(2) = 4$. Hence vertex is $V(2 , 4)$. The zeros of the parabola (i.e. the point where the parabola meets x-axis) are the solutions to $-x^2 +4x = 0$ so $x = 0$ and $x = 4$. Therefore (0,0)and (4,0) lie on the parabola. Also (1,3) and (3,3) lie on the parabola.

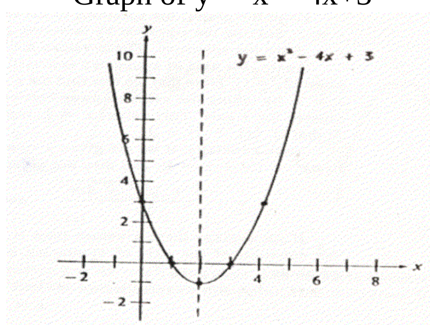
Graph of $y = - x^2 + 4x$



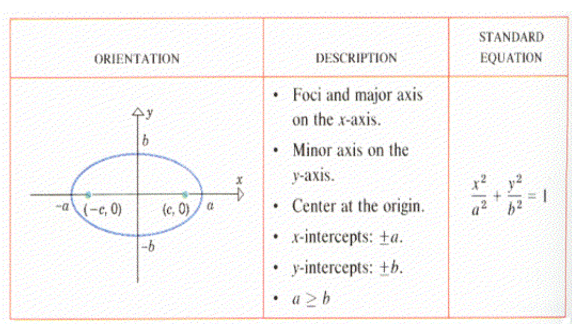
Example $y = x^2 - 4x + 3$

Solution: Since a = 1 > 0, parabola is opening upward.Vertex occurs at $x = -b/2a = (4)/2 = 2$.Axis of symmetry is the vertical line $x = 2$. The y co-ordinate of the vertex is $y = (2)^2 - 4(2) + 3 = -1$.Hence vertex is V(2 , -1)The zeros of the parabola (i.e. the point where the parabola meets x-axis) are the solutions to $x^2 - 4x + 3 = 0$, so $x = 1$ and $x = 3$.Therefore (1,0)and (3,0) lie on the parabola. Also (0 ,3) and (4, 3) lie on the parabola.

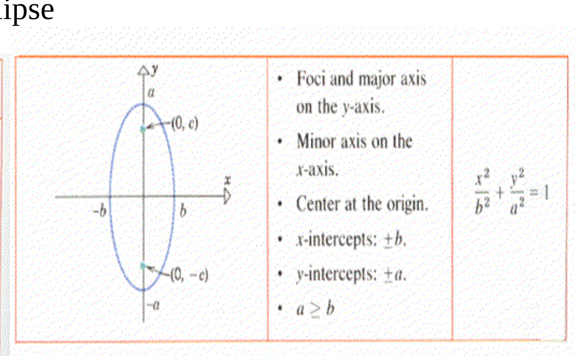
Graph of $y = x^2 - 4x + 3$



Ellipse

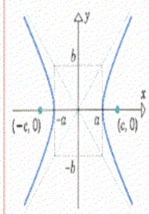


ORIENTATION	DESCRIPTION	STANDARD EQUATION
	• Foci and major axis on the x-axis. • Minor axis on the y-axis. • Center at the origin. • x-intercepts: $\pm a$. • y-intercepts: $\pm b$. • $a \geq b$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



	• Foci and major axis on the y-axis. • Minor axis on the x-axis. • Center at the origin. • x-intercepts: $\pm b$. • y-intercepts: $\pm a$. • $a \geq b$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Hyperbola

ORIENTATION	DESCRIPTION	STANDARD EQUATION	ASYMPTOTE EQUATIONS
	• Foci on the x-axis. • Conjugate axis on the y-axis. • Center at the origin.	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$y = \frac{b}{a}x$ $y = -\frac{b}{a}x$

Home Assignments:

In this lecture we recall some basic geometrical concepts which are prerequisite for this course and you can find all these concepts in the chapter # 12 of your book Calculus By Howard Anton.