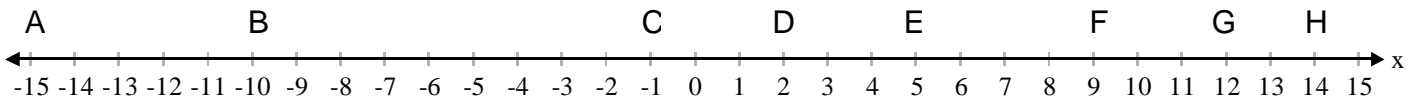# Absolute Value
Geometrically, Algebraically and Technologically

ABSOLUTE VALUE IS A DISTANCE AND A DISTANCE IS ALWAYS POSITIVE!

Using the number line below, find the distances for problems 1 - 4.
Show work or tell in words how you solved those distance questions.
Reminder: In geometry, a distance is indicated by 2 capital letters written side-by-side with no line, segment or ray symbol above them.

```
A         B                        C     D     E          F     G   H
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--> x
-15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

1.) Find AH
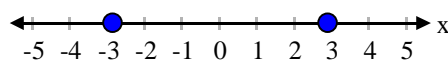
2.) Find GB.

3.) Find BC + EF.

4.) Find DF - CD.

GEOMETRIC INTERPRETATION OF ABSOLUTE VALUE

I)   Write out the "distance sentence."

II)   Graph each of the following on a separate x-axis.

III) Write answer under graph.   The answers are boxed.

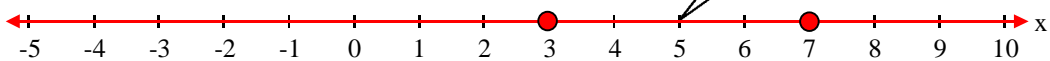Example 1: $|X| = 3$ What numbers are 3 away from zero? Answer: ± 3

```
<---|--|--●--|--|--|--|--●--|--|--> x
   -5 -4 -3 -2 -1  0  1  2  3  4  5
```

$$\boxed{X = -3 \ \text{or} \ X = 3}$$

Example 2: $|X - 5| = 2$

[Note: You use the opposite sign of the number inside the absolute value symbols.]

What numbers are 2 away from +5?   Answer: 3, 7

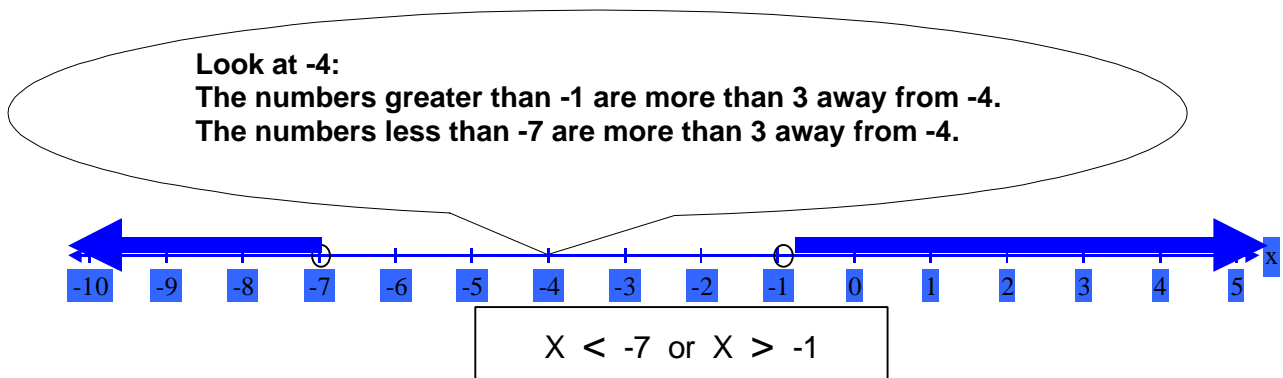Look at **5** on the line: 3 is 2 units to the left of 5 ; 7 is 2 units to the right of 5.

```
<---|----|----|----|----|----|----|----●----|----|----●----|----|---> x
   -5   -4   -3   -2   -1    0    1    2    3    4    5    6    7    8    9   10
```

$$\boxed{X = 3 \ \text{or} \ x = 7}$$

1

Absolute Value
Geometrically, Algebraically and Technologically

Inequality Examples

I)   Write out the "distance sentence.
II)   Graph each of the following on a separate x-axis.
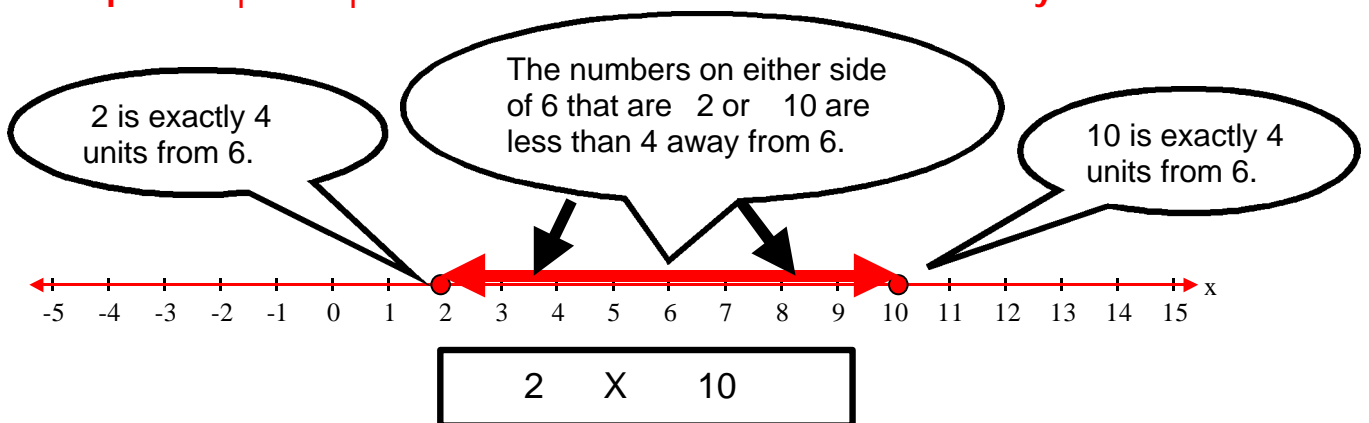III)   Write answer under graph.   The answers are boxed.

Example 3: |X + 4| > 3. What numbers are MORE than 3 away from ⁻4?

Look at -4:
The numbers greater than -1 are more than 3 away from -4.
The numbers less than -7 are more than 3 away from -4.

X < -7 or X > -1

All numbers to the right of ⁻1 are 3 or more away from ⁻4 and all numbers to the left of ⁻7 are 3 or more to the left of ⁻4.

Both ⁻1 and ⁻ 7 are open circles on the number line because there are no = signs below the greater than (>) symbol.

Example 4: |X - 6| ≤ 4 What numbers are 4 or less away from 6?

2 is exactly 4 units from 6.

The numbers on either side of 6 that are 2 or 10 are less than 4 away from 6.
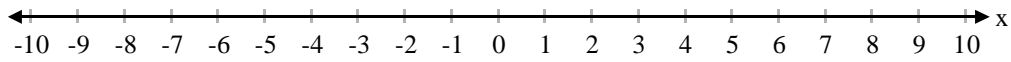
10 is exactly 4 units from 6.

2   X   10

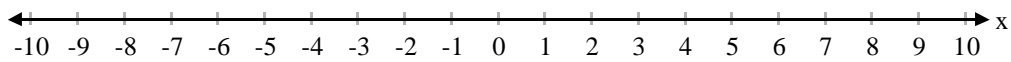Exercises 5 - 11 follow on the next 2 pages. Do them geometrically using the following steps:
I)   Write out the "distance sentence" in the blank provided.
II)   Graph on the x-axis provided.
III)   Write answer in the space under the graph.

5.) $|X| = 5$ _____

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──► x
 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8  9 10
```

_____

6.) $|X - 2| = 6$ _____

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──► x
 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8  9 10
```

_____

7.) $|X + 5| = 2$_____

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──► x
 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8  9 10
```

_____

8.) $|X - 2| > 6$ _____

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──► x
 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8  9 10
```

_____

9.) $|X - 4|$ 3 _____

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──► x
 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8  9 10
```

_____

10.) $|X + 5|$ 2 _____

```
◄─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──► x
 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8  9 10
```

_____

3

11.) $|x+4| = -3^1$ _____

_____
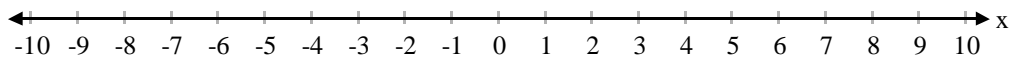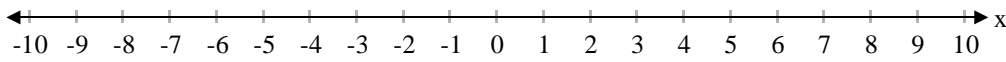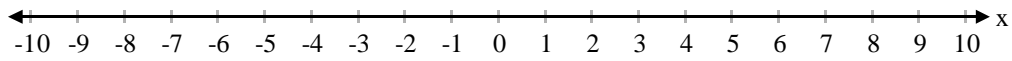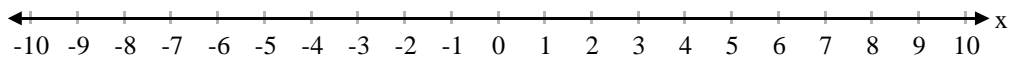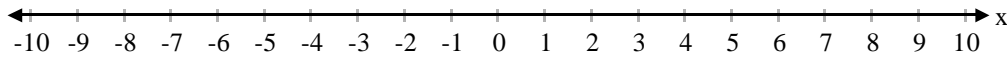
[1] **Hint**: Distance is always positive.

The Algebra of Absolute Value

If the x term of the absolute value quantity has a numerical coefficient (number in front of a variable) other than 1, it is probably better and certainly easier to use algebra to find the distances. All 3 cases of absolute value equations break down into algebraic equations. The absolute value symbols are dropped and each case gives two answers except when the distance is zero or is negative. If the distance is zero, there is only one answer. If the distance is negative, there is no solution.

Case 1: $|Q| = a$

$Q = -a$ OR $Q = a$

Disjunction: **OR**
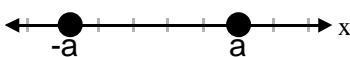
Graph consists of two distinct points and are solid circles. It's an OR graph.

Case 2: $|Q| \gtreqless a$

$Q \lesseqgtr -a$ OR $Q \gtreqless a$

Disjunction: **OR**
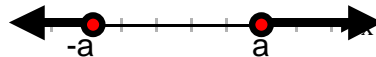
,
circles are solid; if **>** or **<**, circles are open. It's an OR graph.

Case 3: $|Q| \lesseqgtr a$

$-a \lesseqgtr Q \lesseqgtr a$

Conjunction: **AND**

) circles with the area between them shaded. It's a BETWEEN graph.

Examples follow. We will solve the equations on this paper. Then we will verify our geometric and algebraic results with the graphing calculator and graph the calculator screens on the graphs provided.

4

Using your **TI-83+ Calculator**, enter the equation into the equation editor, the y = button, as follows:

1) Y_1= Math, NUM, 1: abs (Q).   The **abs** means absolute value and **Q** is the absolute value quantity that is put into the parentheses.

2) Y_2 = **a**. The a is the constant term or distance; Y_2 = a is a horizontal line.

3) Set a suitable window.

4) Graph
A suitable calculator window
(Friendly Window):

```
WINDOW
 Xmin=-9.4
 Xmax=9.4
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

Calculator Screens for graphing

The y= key

```
Plot1 Plot2 Plot3
\Y1=abs(■
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

The Math key has abs as 1: under NUM

```
MATH NUM CPX PRB
1:abs(
2:round(
3:iPart(
4:fPart(
5:int(
6:min(
7↓max(
```

Examples using "Q" Talk with Algebra and Technology.

Case 1: an "or" case

Case 1:

$$|Q| = a$$

$$|X - 5| = 2$$

$$Q = -a \text{ OR } Q = a$$

X - 5 = -2   or   X - 5 = 2
   + 5 = +5   +5 = +5
X   = 3   or   X   = 7

```
Y1=abs(X-5)
         Y = 2

                    3        7

X=7          Y=2
```
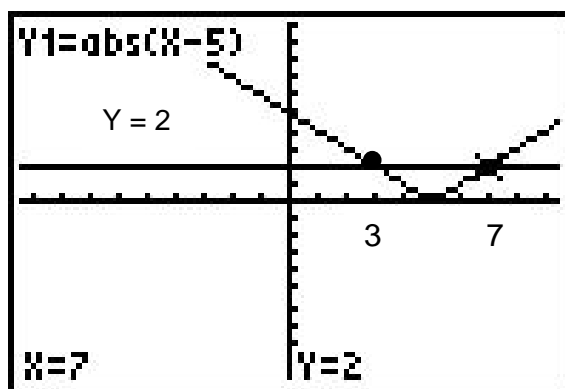
The absolute value graph intersects the line, y = 2, at x = 3 or at x = 7.

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Case 2: an "or" inequality case

$$|Q| > a$$

$$|X + 4| > 3$$

$$Q < -a \quad \text{OR} \quad Q > a$$

X + 4 < -3	or	X + 4 > 3
- 4 = -4		- 4 = -4
X < -7	or	X > -1

Y1=abs(X+4)   Y = 3
-7   -4   -1
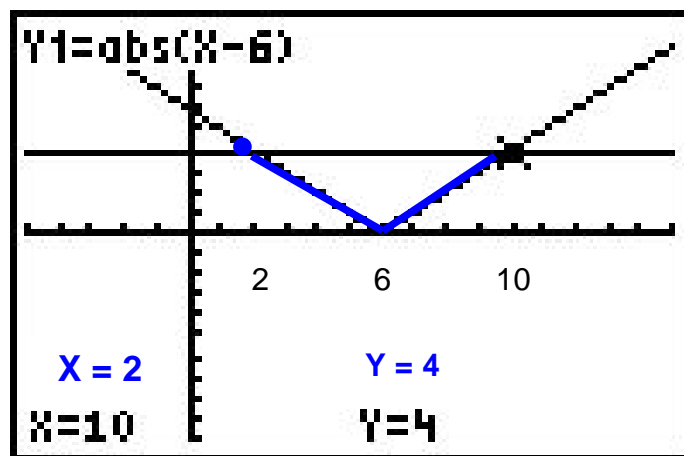X=-7   Y=3
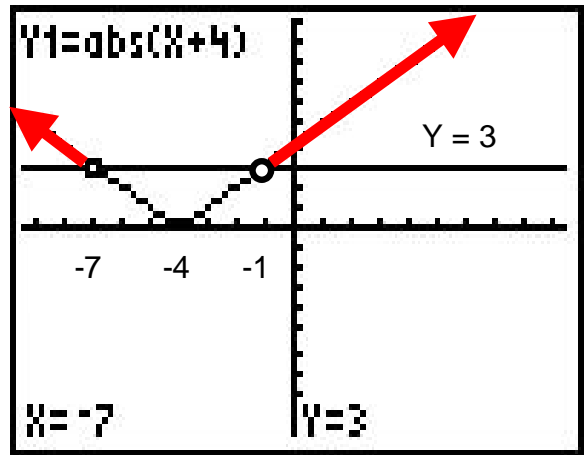
The absolute value graph, |x + 4| is **above** (greater than) the line y = 3 when x < -7 (to the left of -7) or when x > -1(to the right of -1).

Case 3: an "and" inequality case

Case 3: $|Q| \le a$

$$|X - 6| \quad 4$$

$$-a \le Q \le a$$

-4	X - 6	4
6 =	6 =	6
2	X	10

Y1=abs(X-6)
2   6   10
X = 2   **Y = 4**
X=10   Y=4

The absolute value graph, |X - 6|, is **equal** to 4 (intersects the line, y = 4) at the points x = 2 and x = 10 and is less than (**under**) the line y = 4 between 2 and 10.
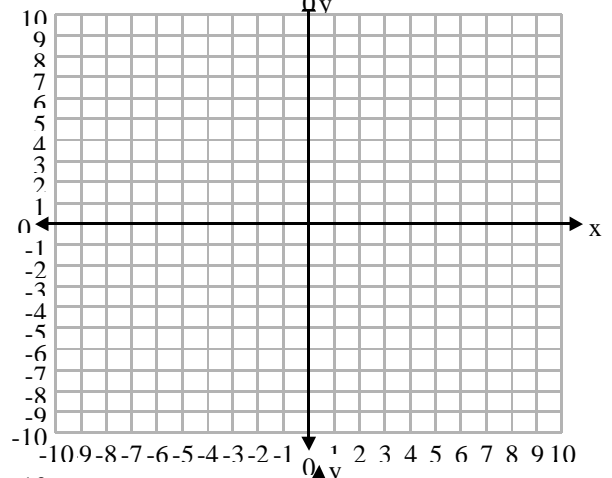
Exercises - These are the same problems that were graphed earlier. Now we will solve them algebraically, graph on the calculator, sketch the calculator screens and compare the work here with the results on pages 3 and 4.
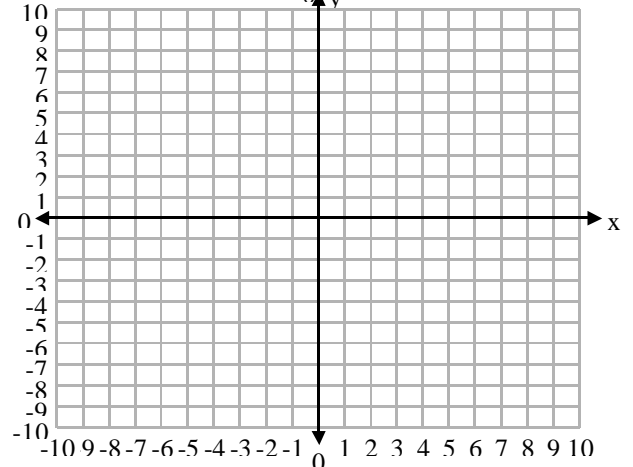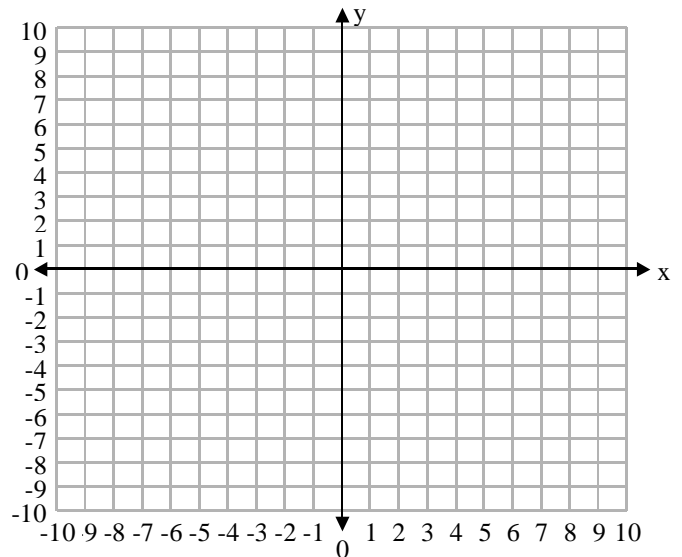
Case 1: an "or" case

6.) $\left| x - 2 \right| = 6$
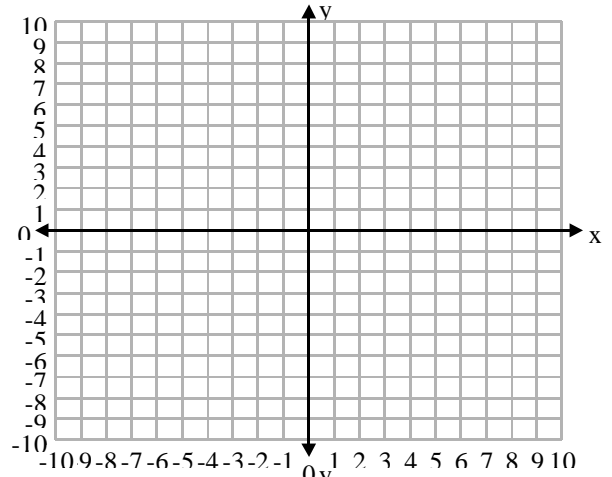
6.a) $\left| 3x + 2 \right| > 5$

7.) $\left| x + 5 \right| = 2$

Case 2: The -*OR*- case

8.) $|x-2|>6$

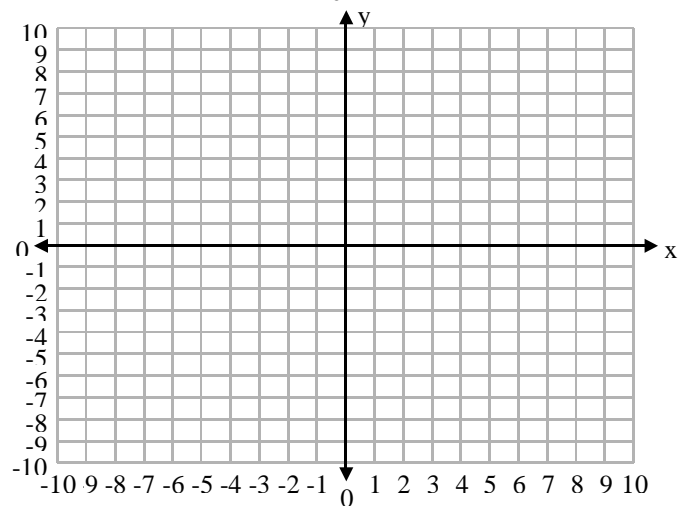**9.a)** $|x-4|\geq 3$
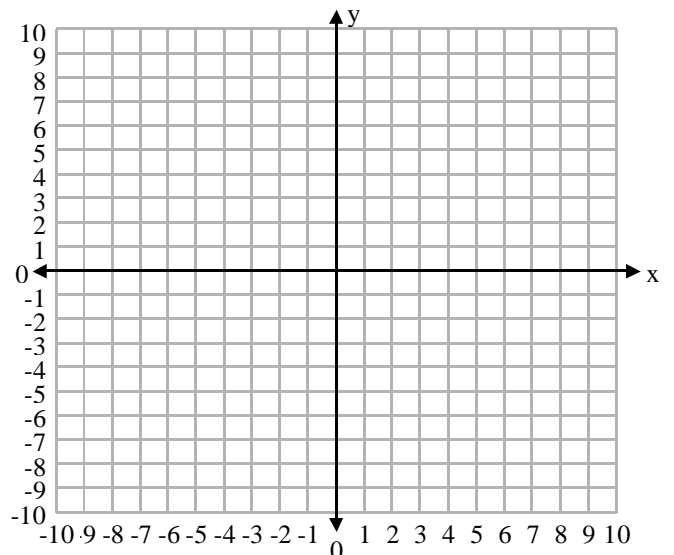
9.b) $|4x+3|\geq 2$

Algebra will be easier here as well since there is a coefficient of 4 in front of the x.
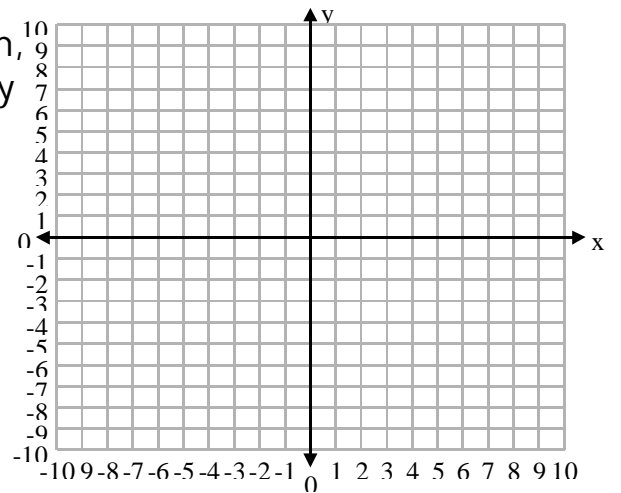
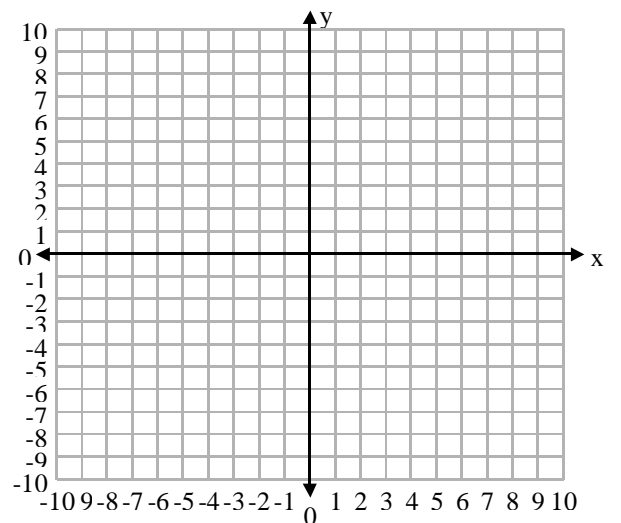### Case 3: The -*AND*- case

10.a) $|x+5| < 2$
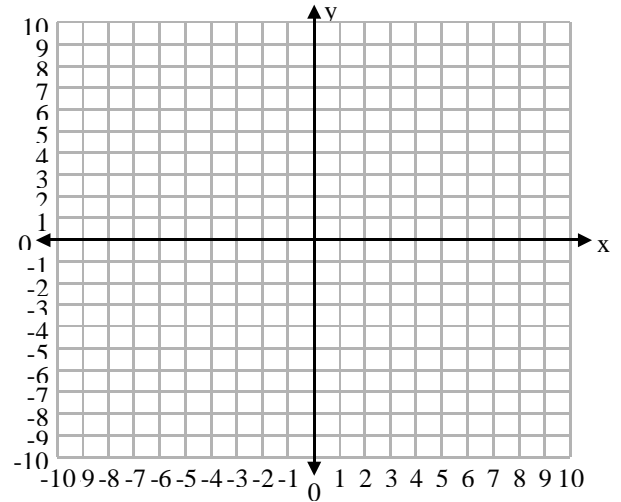
10.b) $|2x-4| \le 6$

Since there is a coefficient of 2 on the x term, it will be much easier to do this algebraically rather than geometrically.

11.) $|x + 4| = {}^-3$
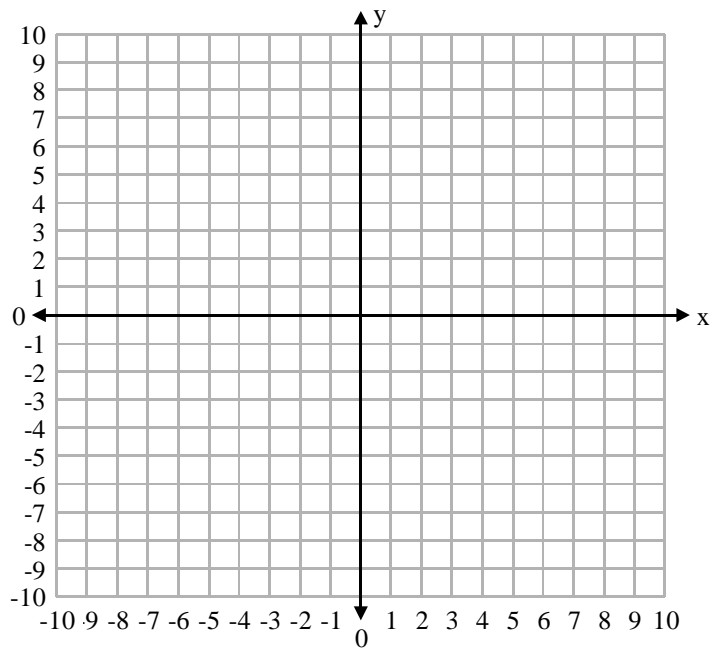
<u>Making Connections</u>

x_1, y_1 x_2, y_2

12.) Using graph at below at the right, plot A(-3,1) and B(5,4).

13.) Find AB geometrically.

14.) Find AB using distance formula.

15.) Find the slope of AB.

10

<h1 style="text-align:center">Absolute Value
Geometrically, Algebraically and Technologically</h1>

Vienna VISIT-ME-2002 Presentation
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e-mail: gaynixon@juno.com

REFERENCES & RESOURCES & REASONS

1) Four types of software were used in the preparation of this paper:

a) Microsoft Word 97
b) MathType Version 5.0 - www.dessci.com or info@dessci.com
c) HandyGraph Version 1.0 - www.handygraph.com
d) TI-83 Plus Graph Link for Windows, Version 2.0 - www.education.ti.com

2) The best math teacher I ever had introduced me to some of these ideas:
   Bob Killingstad
   Everett Community College
   2000 Tower Street, Everett, WA
   Everett, WA 98201-1390
   425-388-9100
   http://www.evcc.ctc.edu/

3) The inspiration to clarify this troublesome topic was encouraged by many
confused tutees I have worked with in my many years of tutoring. All who have
studied absolute value under my tutelage have complimented my explanations
and told me their notes were worth keeping. I intend to continue using this
method with many future tutees.

4) I have intentionally left out cases involving inequalities that lie completely above or below the horizonal line like $|x - 8| > -5$ or $-|x + 7| < 6$. The solution in both cases would be all reals.   However, the length of this paper had to be controlled and these types can be discussed with the students after the concepts presented here are understood.

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