Solution

Extreme Points of $5-3\sin(x)$: Minimum$\left(\frac{\pi}{2}+2\pi n, 2\right)$, Maximum$\left(\frac{3\pi}{2}+2\pi n, 8\right)$

Steps

First Derivative Test definition

Suppose that $x=c$ is a critical point of $f(x)$ then,
If $f'(x)>0$ to the left of $x=c$ and $f'(x)<0$ to the right of $x=c$ then $x=c$ is a local maximum.
If $f'(x)<0$ to the left of $x=c$ and $f'(x)>0$ to the right of $x=c$ then $x=c$ is a local minimum.
If $f'(x)$ is the same sign on both sides of $x=c$ then $x=c$ is neither a local maximum nor a local minimum.

Find the critical points: $x=\frac{\pi}{2}+2\pi n, x=\frac{3\pi}{2}+2\pi n$ *Hide Steps*

Critical point definition

Critical points are points where the function is defined and its derivative is zero or undefined

Find where $f'(x)$ is equal to zero or undefined

$f'(x)=-3\cos(x)$ *Hide Steps*

$\frac{d}{dx}(5-3\sin(x))$

Apply the Sum/Difference Rule: $(f\pm g)'=f'\pm g'$

$=\frac{d}{dx}(5)-\frac{d}{dx}(3\sin(x))$

$\frac{d}{dx}(5)=0$ *Hide Steps*

$\frac{d}{dx}(5)$

Derivative of a constant: $\frac{d}{dx}(a)=0$

$=0$

$\frac{d}{dx}(3\sin(x))=3\cos(x)$ *Hide Steps*

$\frac{d}{dx}(3\sin(x))$

Take the constant out: $(a\cdot f)'=a\cdot f'$

$=3\frac{d}{dx}(\sin(x))$

Apply the common derivative: $\frac{d}{dx}(\sin(x))=\cos(x)$

$=3\cos(x)$

$=0-3\cos(x)$

Simplify

$=-3\cos(x)$

Solve $-3\cos(x)=0$: $x=\frac{\pi}{2}+2\pi n, x=\frac{3\pi}{2}+2\pi n$ *Hide Steps*

$-3\cos(x)=0$

Divide both sides by -3

$\frac{-3\cos(x)}{-3}=\frac{0}{-3}$

Simplify

$\cos(x)=0$

General solutions for $\cos(x)=0$

$x=\frac{\pi}{2}+2\pi n,\ x=\frac{3\pi}{2}+2\pi n$

$x=\frac{\pi}{2}+2\pi n,\ x=\frac{3\pi}{2}+2\pi n$

$x=\frac{\pi}{2}+2\pi n,\ x=\frac{3\pi}{2}+2\pi n$

Identify critical points not in $f(x)$ domain

Domain of $5-3\sin(x)$: $-\infty < x < \infty$ *Hide Steps*

Domain definition

The domain of a function is the set of input or argument values for which the function is real and defined

The function has no undefined points nor domain constraints. Therefore, the domain is

$-\infty < x < \infty$

All critical points are in domain

$x=\frac{\pi}{2}+2\pi n,\ x=\frac{3\pi}{2}+2\pi n$

Domain of $5-3\sin(x)$: $-\infty < x < \infty$ *Hide Steps*

Domain definition

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Periodicity of $5-3\sin(x)$: 2π *Hide Steps*

$\sin(x)$ base periodicity is 2π

$=2\pi$

Combine the critical point(s): $x=\frac{\pi}{2}+2\pi n,\ x=\frac{3\pi}{2}+2\pi n$ with the period: $2\pi n\le x<2\pi+2\pi n$

The function monotone intervals are:

$2\pi n\le x<\frac{\pi}{2}+2\pi n,\ \frac{\pi}{2}+2\pi n<x<\frac{3\pi}{2}+2\pi n,\ \frac{3\pi}{2}+2\pi n<x<2\pi+2\pi n$

Check the sign of $f'(x)=-3\cos(x)$ at each monotone interval

Check the sign of $-3\cos(x)$ at $2\pi n\le x<\frac{\pi}{2}+2\pi n$: Negative *Hide Steps*

Evaluate the derivative at a point on the interval. Take the point $x=1+2\pi n$ and plug it into $-3\cos(x)$

$-3\cos(1+2\pi n)$

Since $f(x)$ is periodic, then $f'(x)$ is also periodic: $-3\cos(x+2\pi n)=-3\cos(x)$

$-3\cos(1)$

Refine to a decimal form

$-1.62091\ldots$

Negative

Check the sign of $-3\cos(x)$ at $\frac{\pi}{2}+2\pi n<x<\frac{3\pi}{2}+2\pi n$: Positive *Hide Steps*

Evaluate the derivative at a point on the interval. Take the point $x=3+2\pi n$ and plug it into $-3\cos(x)$

$-3\cos(3+2\pi n)$

Since $f(x)$ is periodic, then $f'(x)$ is also periodic: $-3\cos(x+2\pi n) = -3\cos(x)$

$-3\cos(3)$

Refine to a decimal form

$2.96998\ldots$

Positive

Check the sign of $-3\cos(x)$ at $\frac{3\pi}{2}+2\pi n < x < 2\pi+2\pi n$: Negative

Hide Steps

Evaluate the derivative at a point on the interval. Take the point $x=5+2\pi n$ and plug it into $-3\cos(x)$

$-3\cos(5+2\pi n)$

Since $f(x)$ is periodic, then $f'(x)$ is also periodic: $-3\cos(x+2\pi n) = -3\cos(x)$

$-3\cos(5)$

Refine to a decimal form

$-0.85099\ldots$

Negative

Summary of the monotone intervals behavior

	$2\pi n \le x < \frac{\pi}{2}+2\pi n$	$x=\frac{\pi}{2}+2\pi n$	$\frac{\pi}{2}+2\pi n < x < \frac{3\pi}{2}+2\pi n$	$x=\frac{3\pi}{2}+2\pi n$	$\frac{3\pi}{2}+2\pi n < x < 2\pi+2\pi n$
Sign	$-$	0	$+$	0	$-$
Behavior	Decreasing	Minimum	Increasing	Maximum	Decreasing

Plug the extreme point $x=\frac{\pi}{2}+2\pi n$ into $5-3\sin(x) \quad \Rightarrow \quad y=2$

Minimum$\left(\frac{\pi}{2}+2\pi n, 2\right)$

Plug the extreme point $x=\frac{3\pi}{2}+2\pi n$ into $5-3\sin(x) \quad \Rightarrow \quad y=8$

Maximum$\left(\frac{3\pi}{2}+2\pi n, 8\right)$

Minimum$\left(\frac{\pi}{2}+2\pi n, 2\right)$, Maximum$\left(\frac{3\pi}{2}+2\pi n, 8\right)$

Graph

Plotting: $y = 5 - 3\sin(x)$