<u>Solution to the Practice questions of Lecture No. 11 and 12</u>

1) If $\lim_{x\to a} g(x) = L$ exists, then it means that for any $\varepsilon > 0$ $g(x)$ is in the interval ____.

I. $(a-L, a+L)$
II. $(a-\delta, a+\delta)$
III. $(L-\delta, L+\delta)$
IV. $(L-\varepsilon, L+\varepsilon)$ **Correct option is IV**

2) Using epsilon-delta definition, $\lim_{x\to 4} f(x) = 6$ can be written as ______.

I. $|f(x)-6| < \varepsilon$ whenever $0 < |x-4| < \delta$ **Correct option is I**
II. $|f(x)-4| < \varepsilon$ whenever $0 < |x-6| < \delta$
III. $|x-6| < \varepsilon$ whenever $0 < |f(x)-4| < \delta$
IV. $|f(x)-x| < \varepsilon$ whenever $0 < |6-4| < \delta$

3) Using epsilon-delta definition, our task is to find δ which will work for any ____.

I. $\varepsilon < 0$
II. $\varepsilon > 0$ **Correct option is II**
III. $\varepsilon \geq 0$
IV. $\varepsilon \leq 0$

4) Using epsilon-delta definition, $\lim_{x\to 1} f(x) = 2$ can be written as ______.

I. $|x-2| < \varepsilon$ whenever $0 < |f(x)-1| < \delta$
II. $|f(x)-x| < \varepsilon$ whenever $0 < |2-1| < \delta$

III. $\left| f(x)-2 \right| < \varepsilon$ whenever $0 < \left| x-1 \right| < \delta$ **Correct option is III**

IV. $\left| f(x)-2 \right| < \varepsilon$ whenever $0 < \left| x-2 \right| < \delta$

5) Which of the following must hold in the definition of limit of a function?

I. ε greater than zero
II. δ greater than zero
III. both ε and δ greater than zero **Correct option is III**
IV. none of these

Solution 6:

To show that $h(x)=2x^2-5x+3$ is continuous for all real numbers, let's consider an arbitrary real number c. Now, we are to show that

$$\lim_{x\to c} f(x) = f(c)$$

$$\begin{aligned}\lim_{x\to c} h(x) &= \lim_{x\to c}(2x^2-5x+3)\\ &= 2c^2-5c+3\\ &= f(\text{c})\end{aligned}$$

Since, it is continuous on an arbitrary real number we can safely say that the given polynomial is continuous on all the real numbers.

Solution 7:

Given function is

$$f(x)=\begin{cases}-2x+8 & for\ \ x\le 4\\ \dfrac{1}{2}x-2 & for\ \ x>4\end{cases}$$

First of all, we will see if the function is defined at x=4. Clearly,

$$\begin{aligned}\text{f}(4) &= -2(4)+8\\ &= -8+8=0\end{aligned}$$

So, yes the function is defined at x =4.
Now, let's check the limit of the function at x =4

$$\begin{aligned}\lim_{x\to 4^-} f(\text{x}) &= \lim_{x\to 4^-}(-2\,\text{x}+8)\\ &= 0\end{aligned}$$

$$\begin{aligned}\lim_{x\to 4^+} f(\text{x}) &= \lim_{x\to 4^-}\left(\frac{1}{2}x-2\right)\\ &= 0\end{aligned}$$

Since the left hand side limit and the right hand side limits exist and are equal so, the limit of the given function exist at x =4. Also,

$$\lim_{x\to 4} f(x) = f(4)$$

Hence, the function is continuous on the given point.

Solution 8:

Given function is

$$g(x) = \begin{cases} x+4 & if \quad x<1 \\ 2 & if \ 1 \le x < 4 \\ -5+x & if \quad x \ge 4 \end{cases}$$

First of all, we will see if the function is defined at x=4. Clearly,

$$g(4) = -5+4$$
$$= -1$$

So the function is defined at x =4.

Now, let's check the limit of the function at x =4.

$$\lim_{x\to 4^-} g(x) = \lim_{x\to 4^-}(2)$$
$$= 2$$
$$\lim_{x\to 4^+} g(x) = \lim_{x\to 4^+}(-5+x)$$
$$= -1$$

Since the left hand side limit is not equal to the right hand side limit, therefore, the limit of the given function does not exist at x =4 and so the function is not continuous on the given point.

Solution 9:

The given function is

$$f(\text{x}) = |x+3|$$

Using the method of finding the limit of composite functions, we can write it as

$$\lim_{x\to 3} f(\text{x}) = \lim_{x\to 3}|x+3|$$
$$= \left|\lim_{x\to 3}(x+3)\right|$$
$$= 6$$

Also,

$$f(3) = |3+3|$$
$$= |6| = 6$$

Since,

$$\lim_{x\to 3} f(x) = f(3)$$

Therefore, the given function is continuous at x=3.

Solution 10:

The given function is

$$f(\text{x}) = \begin{cases} \dfrac{9-x^2}{3-x} & if \quad x \neq 3 \\ 4 & if \quad x = 3 \end{cases}$$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{9-x^2}{3-x}$$

$$= \lim_{x \to 3} \frac{(3-x)(3+x)}{3-x}$$

$$= \lim_{x \to 3} (3+x) = 6$$

$$f(3) = 4$$

Clearly,

$$\lim_{x \to 3} f(x) \neq f(3)$$

Therefore, the given function is not continuous at x=3.