

Solution 1:

Given function is $f(x) = x^2 - 6x + 10$

$$f'(x) = 2x - 6$$

$$f''(x) = 2 > 0$$

Since the second derivative is greater than zero for all values of x , so the given function is concave up on the interval $(-\infty, \infty)$ and it is concave down nowhere.

Solution 2:

The given function is $f(x) = x^3 + 3x^2$

$$f'(x) = 3x^2 + 6x$$

$$f''(x) = 6x + 6$$

For concave up

$$f''(x) = 6x + 6 > 0$$

$$6x > -6$$

$$x > -1$$

So, the given function is concave up on $(-1, \infty)$

For concave down

$$f''(x) = 6x + 6 < 0$$

$$= 6x < -6$$

$$= x < -1$$

So, the given function is concave down on $(-\infty, -1)$.

Solution 3:

It is given that $f'(x) = 1 + 4x$. The function will be increasing on all the values of x where first derivative is greater than zero. That is

$$f'(x) = 1 + 4x > 0$$

$$4x > -1$$

$$x > -\frac{1}{4}$$

Thus, the given function is increasing on $(-\frac{1}{4}, \infty)$.

The function will be decreasing on all the values of x where the first derivative is less than zero.
That is

$$\begin{aligned}f'(x) &= 1 + 4x < 0 \\4x &< -1 \\x &< -\frac{1}{4}\end{aligned}$$

Thus, the given function is decreasing on $(-\infty, -\frac{1}{4})$.

Solution 4:

It is given that $f'(t) = 2t - 2$. The function will be increasing on all the points where the first derivative is greater than zero. That is

$$\begin{aligned}f'(t) &= 2t - 2 > 0 \\2t &> 2 \\t &> 1\end{aligned}$$

Thus, the given function is increasing on $(1, \infty)$

The given function will be decreasing on all the points where the first derivative is less than zero.
That is

$$\begin{aligned}f'(t) &= 2t - 2 < 0 \\2t &< 2 \\t &< 1\end{aligned}$$

Thus, the given function is decreasing on $(-\infty, 1)$.

Solution 5:

The given function is $f(x) = (4 - x)(x + 4)$

$$\begin{aligned}f(x) &= (4 - x)(x + 4) \\&= 4x + 16 - x^2 - 4x \\&= 16 - x^2 \\f'(x) &= -2x \\f''(x) &= -2 < 0\end{aligned}$$

Since the second derivative is less than zero for all the values of x therefore, the given function is concave down on $(-\infty, \infty)$ and it is not concave up anywhere.