

### Solution to Practice Exercise For Lecture 7

**Q1.** Consider functions  $f(x) = (x-2)^3$  and  $g(x) = \frac{1}{x^2}$ . Find the composite function  $(f \circ g)(x)$  and also find the domain of this composite function.

Solution.

Here  $f(x) = (x-2)^3$  and  $g(x) = \frac{1}{x^2}$ , then from the definition of composition of functions, we have

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f\left(\frac{1}{x^2}\right) \\ &= \left(\frac{1}{x^2} - 2\right)^3 \end{aligned}$$

If we put  $x = 0$  in the above function, then the function becomes undefined, so

Domain of  $f \circ g(x) = (-\infty, 0) \cup (0, +\infty)$

**Q2.** Let  $f(x) = x+1$  and  $g(x) = x-2$ . Find  $(f+g)(2)$ .

Solution. From the definition,

$$\begin{aligned} (f+g)(x) &= f(x) + g(x) = x+1 + x-2 \\ &= 2x-1 \end{aligned}$$

Hence, if we put  $x = 2$ , we get

$$(f+g)(2) = 2(2) - 1 = 3$$

**Q3.** Let  $f(x) = x^2 + 5$  and  $g(x) = 2\sqrt{x}$ . Find  $(g \circ f)(x)$ . Also find domain of  $(g \circ f)(x)$ .

Solution.

By definition,

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(x^2 + 5) \\ &= 2\sqrt{x^2 + 5}\end{aligned}$$

$$\text{Domain of } f(x) = (-\infty, +\infty)$$

$$\text{Domain of } g(x) = [0, +\infty)$$

Hence,

$$\text{Domain of } g \circ f(x) = [-\infty, +\infty)$$

**Q4.** Given  $f(x) = \frac{3}{x-2}$ , and  $g(x) = \sqrt{\frac{1}{x}}$

Find the domain of these functions. Also find the intersection of their domains.

**Solution.**

Here  $f(x) = \frac{3}{x-2}$ , so

$$\text{domain of } f(x) = (-\infty, 2) \cup (2, +\infty)$$

Now consider  $g(x) = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}$

$$\text{Domain of } g(x) = (0, +\infty).$$

Also, intersection of domains will be:

$$\text{domain of } f(x) \cap \text{domain of } g(x) = (0, 2) \cup (2, +\infty)$$

**Q5.** Given  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{2}{x-2}$ , find  $(f - g)(3)$ .

**Solution.**

$$(f - g)(x) = f(x) - g(x)$$

$$= \frac{1}{x^2} - \frac{2}{x-2}$$

$$(f - g)(3) = \frac{1}{9} - \frac{2}{1} = \frac{1-18}{9} = \frac{-17}{9}$$