Solution of Practice Questions for Lecture #38 to 40

Question#1

Find the work done by the force 500*x* if an object moves in the positive direction over the interval [0.16, 0.19]?

Solution:

Here

F(x) =500x;   [0.16, 0.19]

$W = \int_{0.16}^{0.19} 500x\,dx$ since $W = \int_{a}^{b} F(x)\,dx$

$= \left|\frac{500\,x^2}{2}\right|_{0.16}^{0.19}$

$= \frac{500}{2}\left|x^2\right|_{0.16}^{0.19}$

$=250[(0.19)^2-(0.16)^2]$

=2.625 J

Question#2

Find the spring constant if a force took 1800 *J* of work to stretch a spring from its natural length of 5*m* to a length of 8*m*?

Solution:

The work done by F is W=$\int_0^3 F(x)dx$

$1800 = \int_0^3 Kxdx$

$= k\int_0^3 xdx$

$= k\left|\frac{x^2}{2}\right|_0^3$

$= \frac{9k}{2}$

3600=9k

or k=400

Question#3

Evaluate the improper integral:

$$\int_{-\infty}^{0} \frac{1}{(2x+1)^3}\,dx$$

Solution:

$$\int_{-\infty}^{0} \frac{1}{(2x+1)^3}\,dx$$

$$= \lim_{t\to\infty} \int_{t}^{0} \frac{1}{(2x+1)^3}\,dx$$

$$= \lim_{t\to\infty} \frac{1}{2} \int_{t}^{0} \frac{1}{(2x+1)^3}\,(2dx)$$

$$= \lim_{t\to\infty} \frac{1}{2} \int_{t}^{0} (2x+1)^{-3}(2dx)$$

$$= \lim_{t\to\infty} \frac{1}{2} \left|\frac{(2x+1)^{-2}}{-2}\right|_{t}^{0}$$

$$= \lim_{t\to\infty} \frac{-1}{4}\ |(2x+1)^{-2}|_{t}^{0}$$

$$= \frac{-1}{4} \lim_{t\to\infty}\ [1-(2t+1)^{-2}]$$

$$= \frac{-1}{4} \lim_{t\to\infty}\ [1-\frac{1}{(2t+1)^2}]$$

$$= \frac{-1}{4}[1-0]$$

$$= \frac{-1}{4}$$

Question#4

Solve the improper integral:

$$\int_1^5 \frac{1}{(x-2)^{\frac{2}{3}}} dx$$

Solution:

$$\int_1^5 \frac{1}{(x-2)^{\frac{2}{3}}} dx$$

$$= \int_1^2 \frac{dx}{(x-2)^{\frac{2}{3}}} + \int_2^5 \frac{dx}{(x-2)^{\frac{2}{3}}}$$

$$= \int_1^2 (x-2)^{-\frac{2}{3}} dx + \int_2^5 (x-2)^{-\frac{2}{3}} dx$$

$$= 3 \left|(x-2)^{\frac{1}{3}}\right|_1^2 \quad +3 \left|(x-2)^{\frac{1}{3}}\right|_2^5$$

$=3[0- (-1)^{1/3}]+3[(3)^{1/3}-0]$

$=3[(3)^{1/3}-(-1)^{1/3}]$

$=3[\ \sqrt[1/3]{3} \quad - \quad \sqrt[1/3]{-1}\ ]$

Question#5

Use L‘Hopital's rule to evaluate the limit:

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{Cos2x + 1}$$

Solution:

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin x - 1}{Cos2x + 1} \qquad \frac{0}{0} form$$

By L'Hopital's rule

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{-2\sin 2x} \qquad \frac{0}{0}\ form$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{-2\sin 2x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{-4\sin x\cos x} \qquad since\ \ \sin 2x = 2\sin x\cos x$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{-4\sin x}$$

$$= \frac{-1}{4}$$

Question#6

Evaluate the limit:

$$\lim_{x\to\infty} \sqrt{x^2 - 5x} - x$$

Solution:

$$\lim_{x\to\infty} \sqrt{x^2 - 5x} - x$$

$$= \lim_{x\to\infty} \frac{\sqrt{x^2 - 5x} - x}{\sqrt{x^2 - 5x} + x} \times \sqrt{x^2 - 5x} + x$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2-5x})^2 - x^2}{\sqrt{x^2-5x}+x}$$

$$= \lim_{x\to\infty} \frac{x^2-5x-x^2}{\sqrt{x^2-5x}+x}$$

$$= \lim_{x\to\infty} \frac{-5x}{\sqrt{x^2-5x}+x}$$

$$= \lim_{x\to\infty} \frac{-5x}{\sqrt{x^2(1-\frac{5}{x})}+x}$$

$$= \lim_{x\to\infty} \frac{-5x}{x\sqrt{(1-\frac{5}{x})}+x}$$

$$= \lim_{x\to\infty} \frac{-5x}{x(\sqrt{\left(1-\frac{5}{x}\right)}+1)}$$

$$= \lim_{x\to\infty} \frac{-5}{\sqrt{\left(1-\frac{5}{x}\right)}+1}$$

$$= \frac{-5}{2}$$
