Solution of Practice Exercise For Lecture 22

Q1. Find the vertical asymptotes for the function $f\left(x\right)=\dfrac{x+4}{x^2-25}$.

Solution.

The vertical asymptotes occur at the points where $f\left(x\right)\to\pm\infty$ i.e $x^2-25=0$

$x^2-25=0$

$\Rightarrow x=\pm5$

Thus vertical asymptotes at $x=\pm5$

Q2. Find the horizontal asymptotes for the function $f\left(x\right)=\dfrac{x+4}{x^2-25}$.

Solution.

Horizontal asymptote can be found by evaluate $\lim\limits_{x\to+\infty} f(\mathrm{x})$

$$\lim_{x\to+\infty} f(\mathrm{x}) = \lim_{x\to+\infty}\frac{x+4}{x^2-25}$$

Divide numerator and denominator by x^2,

$$\lim_{x\to+\infty} f(\mathrm{x}) = \lim_{x\to+\infty}\frac{\dfrac{1}{x}+\dfrac{4}{x^2}}{1-\dfrac{25}{x^2}}=\frac{0+0}{1-0}=0$$

Hence horizontal asymptotes at $y=0$

Q3. If $f\left(x\right)=2x^4-16x^2$, determine all relative extrema for the function using First derivative test.

Solution.

First we will find critical points by putting $f^{/}(\mathrm{x})=0$

$\Rightarrow 8x^3-32x=0$

$\Rightarrow 8x\left(x^2-4\right)=0 \Rightarrow x=0,\ x=\pm2$

Because $f^{/}(x)$ changes from negative to positive around −2 and 2, *f* has a relative minimum at x = −2 and x = 2, . Also, $f^{/}(x)$ changes from positive to negative around 0, and hence, *f* has a relative maximum at x = 0.

Q4. Find the relative extrema of $f(\mathrm{x})=\sin x-\cos x\, on\, [0,2\pi]$ usind 2nd derivative test.

Solution.

First we will find critical points by putting $f^{/}(\mathrm{x})=0$,

$$\Rightarrow \cos x + \sin x = 0$$
$$\Rightarrow \cos x = -\sin x$$
$$\Rightarrow \frac{\sin x}{\cos x} = -1 \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4},\ x = \frac{7\pi}{4}$$

Because $f'(x)$ changes from negative to positive around $x = \frac{7\pi}{4}$, f has a relative minimum at $x = \frac{7\pi}{4}$. Also, $f'(x)$ changes from positive to negative around $x = \frac{3\pi}{4}$, and hence, f has a relative maximum at $x = \frac{3\pi}{4}$.

Using 2nd derivative test

$$f''(x) = -\sin x + \cos x$$
$$f''\left(\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4}$$
$$= -0.707 - 0.707 < 0$$
$$\Rightarrow \text{relative maximum at } \frac{3\pi}{4}$$
$$f''\left(\frac{7\pi}{4}\right) = -\sin\frac{7\pi}{4} + \cos\frac{7\pi}{4}$$
$$= 0.707 + 0.707 > 0$$
$$\Rightarrow \text{relative minimum at } \frac{7\pi}{4}$$

Answer. relative maximum at $x = \frac{3\pi}{4}$, relative minimum at $x = \frac{7\pi}{4}$

Q5. Find the critical points of $f(x) = x^{4/3} - 4x^{1/3}$.

Solution.

For critical point put

$$f'(x) = 0 \Rightarrow \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3} = 0$$

$$\Rightarrow \frac{4}{3}x^{1/3} - \frac{4}{3x^{2/3}} = 0$$

$$\Rightarrow \frac{4}{3}\left(\frac{x-1}{x^{2/3}}\right) = 0$$

$$\Rightarrow \frac{x-1}{x^{2/3}} = 0$$

critical points occur where numerator and denominator is zero. i.e

$$x-1=0,\ x^{2/3}=0$$

$$\Rightarrow x=1, \qquad x=0$$