Solution of Practice Exercise For Lecture 19

Q1. Use implicit differentiation to find $\frac{dy}{dx}$ *if* $2xy = x + y - y^2$.

Solution.

$Here\, 2xy = x + y - y^2.$

Differentiate both sides w.r.t x:

$$\frac{d}{dx}(2xy) = \frac{d}{dx}(x + y - y^2)$$

$$\Rightarrow 2(x\frac{dy}{dx} + y(1)) = 1 + \frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$\Rightarrow 2x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} = 1 - 2y$$

$$\Rightarrow \frac{dy}{dx}(2x + 2y - 1) = 1 - 2y$$

$$\frac{dy}{dx} = \frac{1 - 2y}{2x + 2y - 1}$$

Q2. Use implicit differentiation to find $\frac{dy}{dx}$ *if* $x^5 + 3y^4 - y^3 + x^3y = 4$.

Solution.

Here $x^5 + 3y^4 - y^3 + x^3y = 4.$

Differentiate both sides w.r.t x:

$$\Rightarrow 5x^4 + 12y^3\frac{dy}{dx} - 3y^2\frac{dy}{dx} + (x^3\frac{dy}{dx} + y(3x^2)) = 0$$

$$\Rightarrow 12y^3\frac{dy}{dx} - 3y^2\frac{dy}{dx} + x^3\frac{dy}{dx} = -5x^4 - 3x^2y$$

$$\Rightarrow \frac{dy}{dx}\left(12y^3 - 3y^2 + x^3\right) = -5x^4 - 3x^2y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-5x^4 - 3x^2y}{12y^3 - 3y^2 + x^3}$$

Q3. Use implicit differentiation to find $\frac{dy}{dx}$ *if* $y^2 - 2x = 1 - 2y$.

Solution.

Here $y^2 - 2x = 1 - 2y$

Differentiate both sides w.r.t x:

$$\Rightarrow 2y\frac{dy}{dx} - 2 = -2\frac{dy}{dx}$$

$$\Rightarrow 2y\frac{dy}{dx} + 2\frac{dy}{dx} = 2$$

$$\Rightarrow y\frac{dy}{dx}+\frac{dy}{dx}=1$$

$$\Rightarrow \frac{dy}{dx}(y+1)=1$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{y+1}$$

Q4. Find $\frac{dy}{dx}$ if $x^2+y^2=4$

Solution.

here $x^2+y^2=4$

Differentiate both sides, we get

$$2x+2y\frac{dy}{dx}=0$$

$$\Rightarrow 2y\frac{dy}{dx}=-2x$$

$$\Rightarrow \frac{dy}{dx}=\frac{-2x}{2y}$$

$$\Rightarrow \frac{dy}{dx}=\frac{-x}{y}$$

Q5. If $x^q=y^p$ then find $\frac{dy}{dx}$ in terms of variable "x".

Solution.

Here $x^q=y^p$ ....eq.(1)

Differentiate both sides w.r.t x :

$$qx^{q-1}=py^{p-1}\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{qx^{q-1}}{py^{p-1}} \qquad \text{........eq.(2)}$$

From eq.(1), we have $y=x^{q/p}$, put this value in eq.(2) in place of y, we will have:

$$\frac{dy}{dx}=\frac{qx^{q-1}}{p\left(x^{q/p}\right)^{p-1}}=\frac{qx^{q-1}}{px^{q-\frac{q}{p}}}=\frac{q}{p}x^{q-1-\left(q-\frac{q}{p}\right)}=\frac{q}{p}x^{-1+\frac{q}{p}}$$

Hence,

$$\frac{dy}{dx}=\frac{q}{p}x^{\frac{q}{p}-1}$$