

Solution of Practice Exercise For Lecture 16

Q1. Differentiate $g(t) = \frac{t^2 + 4}{2t}$.

Solution.

Here $g(t) = \frac{t^2 + 4}{2t}$

We will use quotient rule to differentiate, therefore

$$\begin{aligned} g'(t) &= \frac{2t \frac{d}{dt}(t^2 + 4) - (t^2 + 4) \frac{d}{dt}(2t)}{(2t)^2} \\ &= \frac{2t(2t) - (t^2 + 4)(2)}{4t^2} = \frac{4t^2 - 2t^2 - 8}{4t^2} = \frac{2t^2 - 8}{4t^2} = \frac{t^2 - 4}{2t^2} \end{aligned}$$

Q2. Evaluate $\frac{d}{dx}((x+1)(1+\sqrt{x}))$ at $x=9$.

Solution.

We will use product rule to evaluate:

$$\begin{aligned} \frac{d}{dx}((x+1)(1+\sqrt{x})) &= (x+1) \frac{d}{dx}(1+\sqrt{x}) + (1+\sqrt{x}) \frac{d}{dx}(x+1) \\ &= (x+1) \left(\frac{1}{2\sqrt{x}} \right) + (1+\sqrt{x})(1) \\ &= \frac{(x+1)}{2\sqrt{x}} + (1+\sqrt{x}) \end{aligned}$$

put $x=9$, we have,

$$\frac{(9+1)}{2\sqrt{9}} + (1+\sqrt{9}) = \frac{10}{6} + 4 = \frac{10+24}{6} = \frac{34}{6} = \frac{17}{3}$$

Q3. Differentiate the following functions.

(a) $h(x) = (2x+1)(x+\sqrt{x})$.

Solution.

We will use product rule to differentiate:

$$\begin{aligned} \frac{d}{dx}(h(x)) &= (2x+1) \frac{d}{dx}(x+\sqrt{x}) + (x+\sqrt{x}) \frac{d}{dx}(2x+1) \\ &= (2x+1) \left(1 + \frac{1}{2\sqrt{x}} \right) + (x+\sqrt{x})(2) \\ &= (2x+1) \left(\frac{2\sqrt{x}+1}{2\sqrt{x}} \right) + (2x+2\sqrt{x}) \end{aligned}$$

$$\begin{aligned}
&= \frac{4x^{3/2} + 2x + 2\sqrt{x} + 1 + 2\sqrt{x}(2x + 2\sqrt{x})}{2\sqrt{x}} \\
&= \frac{4x^{3/2} + 2x + 2\sqrt{x} + 1 + 4x^{3/2} + 4x}{2\sqrt{x}} \\
&= \frac{8x^{3/2} + 2\sqrt{x} + 6x + 1}{2\sqrt{x}}
\end{aligned}$$

(b) $g(x) = x^{-3}(5x^{-4} + 3)$

Solution.

Here $g(x) = x^{-3}(5x^{-4} + 3) = 5x^{-7} + 3x^{-3}$

$$\begin{aligned}
\frac{d}{dx}(g(x)) &= 5\frac{d}{dx}(x^{-7}) + 3\frac{d}{dx}(x^{-3}) \\
&= 5(-7x^{-8}) + 3(-3x^{-4}) \\
&= -35x^{-8} - 9x^{-4}
\end{aligned}$$

(c) $f(x) = \frac{x^3 + 1}{4x^2 + 1}$

Solution.

Here $f(x) = \frac{x^3 + 1}{4x^2 + 1}$

We will use quotient rule to differentiate:

$$\begin{aligned}
\frac{d}{dx}(f(x)) &= \frac{(4x^2 + 1)\frac{d}{dx}(x^3 + 1) - (x^3 + 1)\frac{d}{dx}(4x^2 + 1)}{(4x^2 + 1)^2} \\
&= \frac{(4x^2 + 1)(3x^2) - (x^3 + 1)(8x)}{(4x^2 + 1)^2} \\
&= \frac{12x^4 + 3x^2 - (8x^4 + 8x)}{(4x^2 + 1)^2} \\
&= \frac{4x^4 + 3x^2 - 8x}{(4x^2 + 1)^2}
\end{aligned}$$