Solution To Practice Exercise For Lecture 23

Q1. Find the minimum value of the function $f(x) = x^3 - 27x + 4$ attains in the interval [4,-4].

Solution.

First of all we have to find critical points by putting $f'(x)$=0

$$f(x) = x^3 - 27x + 4$$
$$f'(x) = 3x^2 - 27$$
$$put\ f'(x) = 0$$
$$\Rightarrow 3x^2 - 27 = 0$$
$$\Rightarrow 3(x^2 - 9) = 0$$
$$\Rightarrow x^2 - 9 = 0 \Rightarrow x = -3, 3$$

now we have points 4,-4, -3 and 3 we will check on all these points

$$f(4) = (4)^3 - 27(4) + 4 = -40$$
$$f(-2) = (-4)^3 - 27(-4) + 4 = 48$$
$$f(3) = (3)^3 - 27(3) + 4 = -50$$
$$f(-3) = (-3)^3 - 27(-3) + 4 = 58$$

So minimum value $= -50$

Q2. Find the maximum value of the function $f(x) = x^3 + 3x^2 - 9x$ attains in the interval [-4, 3].

Solution.

First of all we have to find critical points by putting $f'(x) = 0$

$$f(x) = x^3 + 3x^2 - 9x$$
$$f'(x) = 3x^2 + 6x - 9$$
$$put\ f'(x) = 0$$
$$\Rightarrow 3x^2 + 6x - 9 = 0$$
$$\Rightarrow x^2 + 2x - 3 = 0$$
$$\Rightarrow x^2 + 3x - x - 3 = 0$$
$$\Rightarrow x(x+3) - (x+3) = 0$$
$$\Rightarrow (x-1)(x+3) = 0 \ \Rightarrow x = 1, x = -3$$

now we have points 1,-3, 3 and -4 we will check on all these points

$f(1) = (1)^3 + 3(1)^2 - 9(1) = -5$

$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) = 27$

$f(3) = (3)^3 + 3(3)^2 - 9(3) = 27$

$f(-4) = (-4)^3 + 3(-4)^2 - 9(-4) = 20$

Hence maximum value = $f(-3) = f(3) = 27$

Q3. Find the absolute maximum and absolute minimum values of the function $f(x) = 4 - x^2$ on interval $-3 \le x \le 1$.

Solution.

First we will find critical points:

put $f'(x) = 0$

$\Rightarrow -2x = 0$

$\Rightarrow x = 0$

Now we find value of $f(x)$ at critical point and at the end points of interval :

$f(0) = 4 - (0)^2 = 4$

$f(1) = 4 - (1)^2 = 3$

$f(-3) = 4 - (-3)^2 = -5$

Hence absolute maximum = 4 and absolute minimum=-5

Q4. Find the absolute maximum and absolute minimum values of the function $f(x) = 2 + x$ on interval $-2 \le x \le 2$.

Solution.

First we will find critical points:

Since $f'(x) = 1$, so there is no critical points.

Now we find value of $f(x)$ at the end points of interval :

$f(-2) = 2 - 2 = 0$

$f(2) = 2 + 2 = 4$

Hence absolute maximum = 4 and absolute minimum=0

Q5. Find the maximum and minimum value of the function $f(x) = 3x^4 - 24x^2 + 1$ on the interval $(-\infty, +\infty)$.

Solution.

This is a continuous function on the given interval and

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} (3x^4 - 24x^2 + 1) = +\infty$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (3x^4 - 24x^2 + 1) = +\infty$$

So f has a minimum but no maximum value in the interval $(-\infty, +\infty)$. To find the minimum value put $f^{/}(x) = 0$ i.e

$f'(x) = 12x^3 - 48x = 0 \Rightarrow 12x(x^2 - 4) = 0 \Rightarrow x = 0 \, and \; x = \pm 2$ are the critical points.

At $x = 0$, $f(0) = 1$,

at $x = 2$, $f(2) = 3(2)^4 - 24(2)^2 + 1 = -47$ and

at $x = -2$, $f(-2) = 3(-2)^4 - 24(-2)^2 + 1 = -47$

so minimum value occurs at $x = \pm 2$ and it is equal to '$f(x) = -47$'