Solution To Practice Exercise For Lecture 10

Q1. Evaluate $\lim_{x\to 5}\frac{x-5}{x^2-25}$.

Solution. First we cancel out the zero in denominator by factorization:

$$\lim_{x\to 5}\frac{x-5}{x^2-25}=\lim_{x\to 5}\frac{x-5}{(x-5)(x+5)}=\lim_{x\to 5}\frac{1}{x+5},$$

Now apply limit, we get:

$$\lim_{x\to 5}\frac{1}{x+5}=\frac{1}{10}$$

Q2. Evaluate $\lim_{x\to 2}\frac{x^2-7x+10}{x-2}$.

Solution.

Factorize the numerator in the expression:

$$\lim_{x\to 2}\frac{x^2-7x+10}{x-2}=\lim_{x\to 2}\frac{x^2-5x-2x+10}{x-2}$$

$$=\lim_{x\to 2}\frac{\text{x}(x-5)-2(\text{x}-5)}{x-2}$$

$$=\lim_{x\to 2}\frac{(x-5)(x-2)}{x-2}$$

$$=\lim_{x\to 2}(\text{x}-5)=2-5=-3$$

Q3. Evaluate $\lim_{x\to 3}\frac{3x^3-9x^2+x-3}{x^2-9}$

Solution. First we factorize the numerator and denominator and then apply its limit:

$$\lim_{x\to 3}\frac{3x^3-9x^2+x-3}{x^2-9}=\lim_{x\to 3}\frac{3x^2(\text{x}-3)+1(\text{x}-3)}{(x-3)(\text{x}+3)}$$

$$=\lim_{x\to 3}\frac{(3x^2+1)(\text{x}-3)}{(x-3)(\text{x}+3)}$$

$$=\lim_{x\to 3}\frac{(3x^2+1)}{(\text{x}+3)}$$

$$=\frac{3(3)^2+1}{3+3}=\frac{28}{6}=\frac{14}{3}.$$

Q4. Let $f(\text{x}) = \begin{cases} 3 - x, & x < 2 \\ \dfrac{x}{2} + 1, & x > 2 \end{cases}$

Determine whether $\lim_{x \to 2} f(\text{x})$ exist or not?

Solution. For limit to exist, we must determine whether left-hand limit and right-hand limit at $x = 2$ exist or not. So here we will find right hand and left hand limit.

Right-hand limit at $x = 2$:

$$\lim_{x \to 2^+} f(\text{x}) = \lim_{x \to 2^+} \left(\frac{x}{2} + 1 \right) = \frac{2}{2} + 1 = 1 + 1 = 2$$

Left-hand limit at $x = 2$:

$$\lim_{x \to 2^-} f(\text{x}) = \lim_{x \to 2^+} (3 - x) = 3 - 2 = 1$$

Clearly $\lim_{x \to 2^+} f(\text{x}) \neq \lim_{x \to 2^-} f(\text{x})$, so limit does not exist.

Q5. If $f(\text{x}) = \begin{cases} 3x + 7, & 0 < x < 3 \\ 16, & x = 3 \\ \text{x}^2 + 7, & 3 < x < 6 \end{cases}$, then show that $\lim_{x \to 3} f(\text{x}) = \text{f}(3)$.

Solution.

Here $f(3) = 16$. To find limit at $x = 3$, we have to find the left-hand and right-hand limit at $x = 3$, so:

$$\lim_{x \to 3^+} f(\text{x}) = \lim_{x \to 3^+} \left(x^2 + 7 \right) = 9 + 7 = 16$$

And

$$\lim_{x \to 3^-} f(\text{x}) = \lim_{x \to 3^-} (3x + 7) = 9 + 7 = 16$$

Clearly $\lim_{x \to 3^+} f(\text{x}) = 16 = \lim_{x \to 3^-} f(\text{x})$, so $\lim_{x \to 3} f(\text{x}) = 16$