Mth101: Practice Questions For

Lecture 29, 30 and 31

Question 1:

Find $\int_{2}^{5} f(x)dx$ if $\int_{3}^{2} f(x)dx = -5$, $\int_{3}^{4} f(x)dx = 3$, $\int_{5}^{4} f(x)dx = 4$

Solution:

$$\int_{5}^{4} f(x)dx = 4 \Rightarrow \int_{4}^{5} f(x)dx = -4$$

$$\text{and } \int_{3}^{2} f(x)dx = -5 \Rightarrow \int_{2}^{3} f(x)dx = 5$$

$$\therefore \int_{2}^{5} f(x)dx = \int_{2}^{3} f(x)dx + \int_{3}^{4} f(x)dx + \int_{4}^{5} f(x)dx$$

$$= 5 + 3 - 4$$

$$= 4$$

Question 2:

Express the area of the region below the line $7x + 2y = 25$, above x-axis and between the lines $x = 0$, $x = 4$ as a definite integral. Also express this integral as a limit of the Riemann Sum.

Solution:

$$7x + 2y = 25$$

$$2y = 25 - 7x$$

$$y = \frac{25 - 7x}{2}$$

$$\int_{0}^{4} \frac{25 - 7x}{2} dx$$

$$\int_{0}^{4} \frac{25 - 7x}{2} dx = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \left(\frac{25 - 7x_k^{*}}{2} \right) \Delta x_k$$

Question 3:

Evaluate the integral $\int_1^3 \frac{x^3-1}{x-1}dx$.

Solution:

$$\int_1^3 \frac{x^3-1}{x-1}dx = \int_1^3 \frac{(x-1)(x^2+x+1)}{x-1}dx$$
$$= \int_1^3 (x^2+x+1)dx$$
$$= \frac{x^3}{3}+\frac{x^2}{2}+x\Big|_1^3$$
$$= \left(\frac{27}{3}+\frac{9}{2}+3\right)-\left(\frac{1}{3}+\frac{1}{2}+1\right) = \left(12+\frac{9}{2}\right)-\left(\frac{2+3+6}{6}\right)$$
$$= \left(\frac{24+9}{2}\right)-\frac{11}{6} = \frac{33}{2}-\frac{11}{6} = \frac{99-11}{6} = \frac{88}{6} = \frac{44}{3}$$

Question 4:

Evaluate the definite integral $\int_1^4 \left(3+3\sqrt{x}\right)dx$.

Solution:

$$\int_1^4 \left(3+3\sqrt{x}\right)dx = 3x+3\frac{x^{3/2}}{3/2}\Big|_1^4$$
$$= 3x+2x^{3/2}\Big|_1^4$$
$$= \left(3(4)+2(4)^{3/2}\right)-\left(3(1)+2(1)^{3/2}\right) = \left(12+2(8)\right)-\left(3+2\right) = 28-5 = 23$$

Question5:

Evaluate the integral $\int \frac{1}{\sqrt{x}.(2+\sqrt{x})^3}dx$ by using proper substitution.

Solution:

Let $u = 2 + \sqrt{x}$

$$\Rightarrow du = \frac{1}{2\sqrt{x}}\, dx$$

$$\Rightarrow 2du = \frac{1}{\sqrt{x}}\, dx$$

Putting these values in the given integral, it becomes

$$\int \frac{1}{\sqrt{x}.(2+\sqrt{x})^3}\, dx = 2\int \frac{1}{u^3}\, du =$$

$$= 2\int u^{-3}\, du$$

$$= 2\left|\frac{u^{-3+1}}{-3+1}\right| = \frac{2}{-2u^2}$$

$$= \frac{-1}{\left(2+\sqrt{x}\right)^2} + c$$

Question 6:

Use the Substitution method to express the following definite integrals in terms of the variable ' u ' but do not evaluate the integrals.

i. $\int_0^{\frac{\pi}{2}} e^{\sin t} \cos t \; dt$

ii. $\int_0^1 4t\sqrt{1-t^2} \; dt$

Solution:

i.

Let $u = \sin t$

$$\Rightarrow du = \cos t\, dt$$

$$t = 0 \Rightarrow u = \sin 0 = 0$$

and $t = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = 1$

as t goes from 0 to $\frac{\pi}{2}$

so u goes from 0 to 1

$$\int_0^{\frac{\pi}{2}} e^{\sin t} \cos t \; dt = \int_0^1 e^u \; (du) = \int_0^1 e^u du$$

ii.

Let

$$u = 1 - t^2$$

$$\Rightarrow du = -2\,tdt$$

$$-2du = 4tdt$$

$$t = 1 \Rightarrow u = 1 - (1)^2 = 0$$

and $\quad t = 0 \Rightarrow u = 1 - (0)^2 = 1$

as $\quad$ t goes from 0 to 1

so $\quad$ u goes from 1 to 0

$$\int_0^1 4t\sqrt{1-t^2}\ dt = -2\int_1^0 \sqrt{u}\ du = 2\int_0^1 \sqrt{u}\ du \quad \begin{pmatrix} \because -2\,tdt = du \\ 4tdt = -2\,du \end{pmatrix}$$