MTH101: Practice Questions for Lecture No. 11: Limit: Rigorous Approach Lecture No.12: Continuity

Choose the correct option for the following questions:

1) If $\lim_{x \to a} g(x) = L$ exists, then it means that for any $\varepsilon > 0$ g(x) is in the interval _____.

I. (a-L,a+L)II. $(a-\delta,a+\delta)$ III. $(L-\delta,L+\delta)$ IV. $(L-\varepsilon,L+\varepsilon)$

2) Using epsilon-delta definition, $\lim_{x \to 4} f(x) = 6$ can be written as _____.

I. $|f(x)-6| < \varepsilon$ whenever $0 < |x-4| < \delta$ II. $|f(x)-4| < \varepsilon$ whenever $0 < |x-6| < \delta$ III. $|x-6| < \varepsilon$ whenever $0 < |f(x)-4| < \delta$ IV. $|f(x)-x| < \varepsilon$ whenever $0 < |6-4| < \delta$

3) Using epsilon-delta definition, our task is to find δ which will work for any _____.

I. $\varepsilon < 0$ II. $\varepsilon > 0$ III. $\varepsilon \ge 0$ IV. $\varepsilon \le 0$

4) Using epsilon-delta definition, $\lim_{x \to 1} f(x) = 2$ can be written as _____.

I. $|x-2| < \varepsilon$ whenever $0 < |f(x)-1| < \delta$

II. $|f(x) - x| < \varepsilon$ whenever $0 < |2 - 1| < \delta$ III. $|f(x) - 2| < \varepsilon$ whenever $0 < |x - 1| < \delta$ IV. $|f(x) - 2| < \varepsilon$ whenever $0 < |x - 2| < \delta$

5) Which of the following must hold in the definition of limit of a function?

- I. ε greater than zero
- II. δ greater than zero
- III. both ε and δ greater than zero
- IV. none of these

Question 6:

Show that $h(x) = 2x^2 - 5x + 3$ is a continuous function for all real numbers.

Question 7:

Discuss the continuity of the following function at x = 4

$$f(x) = \begin{cases} -2x+8 & \text{for } x \le 4\\ \frac{1}{2}x-2 & \text{for } x > 4 \end{cases}.$$

Question 8:

Check the continuity of the following function at x = 4

$$g(x) = \begin{cases} x+4 & if \ x < 1 \\ 2 & if \ 1 \le x < 4 \\ -5+x & if \ x \ge 4 \end{cases}$$

Question 9:

Check the continuity of the following function at x = 3

$$f(\mathbf{x}) = |\mathbf{x} + \mathbf{3}|$$

Question 10:

State why the following function fails to be continuous at x=3.

$$f(\mathbf{x}) = \begin{cases} \frac{9 - x^2}{3 - x} & \text{if } x \neq 3\\ 4 & \text{if } x = 3 \end{cases}$$