Practice Questions of Lecture 35 to 37_Solution

Q.1: Find the equation of cylinder whose directrix is $C: x^2 + y^2 = 9$ and having elements parallel to $\vec{n} = [1, -2, 1]$.

Solution:

Since $\vec{n}$ is not parallel to the $yz-$plane, the cylinder exists. We shall derive an equation of the cylinder by definition.

Equation of the line L through $(x_1, y_1, 0)$ and parallel to $\vec{n}$ are

$$L: \begin{cases} x = x_1 + t \\ y = y_1 - 2t \\ z = t \end{cases}$$

$t = z,$

Thus $x_1 = x - t = x - z$

$y_1 = y + 2t = y + 2z$

As $(x_1, y_1, 0)$ lies on the directrix $x^2 + y^2 = 9$, so

$$x_1^2 + y_1^2 = 9$$

Hence, an equation of the cylinder is

$$x_1^2 + y_1^2 = 9$$

$$\Rightarrow (x - z)^2 + (y + 2z)^2 = 9$$

$$\Rightarrow x^2 + y^2 + 5z^2 + 4yz - 2xz - 9 = 0$$

Q.2: Find the equation of right cylinder whose directrix is circle with center (2, 3, 5) and radius 4.

Solution:

Equation of directrix is

$$(x - 2)^2 + (y - 3)^2 + (z - 5)^2 = 4^2$$

$$\Rightarrow x^2 + y^2 + z^2 - 4x - 6y - 10z + 22 = 0$$

The cylinder is right cylinder. Therefore the equation of directrix is also the equation of cylinder.

Q.3: Find the equation of cone whose directrix is $y^2 + z^2 = 1,\ x = 2$ and vertex $A = (0, 0, 0)$.

Solution:

Let $Q(x_1, y_1, z_1)$ be on the directrix. Then

$y_1^2 + z_1^2 = 1,\ x_1 = 2$ ……………..(1)

Let $P(x, y, z)$ be a point on the cone then equation of line AQ are

$$\frac{x}{x_1}=\frac{y}{y_1}=\frac{z}{z_1}$$

$$\Rightarrow \frac{x}{2}=\frac{y}{y_1}=\frac{z}{z_1}$$

$$\Rightarrow y_1=\frac{2y}{x} \; and \; z_1=\frac{2z}{x}$$

Put these in equation (1), we get

$$\left(\frac{2y}{x}\right)^2+\left(\frac{2z}{x}\right)^2=1$$

$$\Rightarrow 4y^2+4z^2=x^2$$

Q.4: Find the traces of the cone $\frac{y^2}{16}+\frac{z^2}{25}=x^2$ in $xy-$plane, $xz-$plane and $yz-$plane.

Solution:

For trace in xy-plane, put z=0, we get:

$$\frac{y^2}{16}=x^2 \Rightarrow y=\pm 4x$$

which is a pair of lines.

For trace in xz-plane, put y=0, we get:

$$\frac{z^2}{25}=x^2 \Rightarrow z=\pm 5x$$

which is a pair of lines.

For trace in yz-plane, put x=0, we get

$$\frac{y^2}{16}+\frac{z^2}{25}=0$$

which is a point.

Q.5: Identify the surface $\frac{(x-2)^2}{2^2}+\frac{(y-4)^2}{5^2}+\frac{(z+1)^2}{3^2}=1$ and find its center and y-intercept.

Solution:

Compare it with general equation of ellipsoid $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}+\frac{(z-l)^2}{c^2}=1$, we readily that the given equation is ellipsoid with center at (2, 4, -1). To find y-intercept, put x = 0 and z = 0.

$$\frac{(0-2)^2}{2^2}+\frac{(y-4)^2}{5^2}+\frac{(0+1)^2}{3^2}=1$$

$$\Rightarrow \frac{4}{4}+\frac{(y-4)^2}{25}+\frac{1}{9}=1$$

$$\Rightarrow \frac{(y-4)^2}{5^2}=-\frac{1}{9}$$

Hence given ellipsoid has no y-intercept.

Q.6: Identify the surface $100x^2+36y^2+225z^2-200x-216y-4500z+22024=0$ and find its center.

Solution:

$$100x^2+36y^2+225z^2-200x-216y-4500z+22024=0$$

$$\Rightarrow 100x^2-200x+36y^2-216y+225z^2-4500z=-22024$$

$$\Rightarrow 100(x^2-2x)+36(y^2-6y)+225(z^2-20z)=-22024$$

$$\Rightarrow 100(x^2-2x+1)+36(y^2-6y+9)+225(z^2-20z+100)=-22024+100+324+22500$$

$$\Rightarrow 100(x-1)^2+36(y-3)^2+225(z-10)^2=900$$

divide both side by 900, we get

$$\Rightarrow \frac{(x-1)^2}{9}+\frac{(y-3)^2}{25}+\frac{(z-10)^2}{4}=1$$

$$\Rightarrow \frac{(x-1)^2}{3^2}+\frac{(y-3)^2}{5^2}+\frac{(z-10)^2}{2^2}=1$$

which is a ellipsoid with center at (1, 3, 10).