What is Calculus ?

Calculus is a rate of change of one quantity with respect to another quantity.

For example,

- The change of distance with respect to time which gives speed of the moving object.
- The flow of water into a tank with respect to time.

There are two mathematicians who played very important role in the development of Calculus:

1) Isaac Newton (English Mathematician)
2) Gottfried Leibniz (French Mathematician)

	
Isaac Newton	Gottfried Leibniz

Number system

N = the set of natural numbers

= { 1, 2, 3, ...}

The ancient man used these numbers naturally in order to COUNT objects, e.g. 3 trees, 7 sheep, 10 mountains etc.

W = the set of whole numbers

= { 0, 1, 2, 3, ... }

Z = the set of integers

= { ... −3, −2, −1, 0, 1, 2, 3, ... }

USES

- The negative numbers were used to solve such equation

$$x + 2 = 0$$

Its solution is $x = -2$

- The negative numbers show the deficit or loss in business, too.

DIGITS

0, 1, 2, 3,…, 9 are called digits.

In the number 489, there are three digits, i.e. 4, 8 and 9.

The number 2.357 has four digits, i.e. 2, 3, 5 and 7.

Rational Numbers

Rational numbers are those numbers which can be written in form of RATIO $\frac{p}{q}$, where p and q are integers and q is non-zero.

The set of rational numbers is denoted by Q.

$$Q = \{ \frac{p}{q} \mid p, q \in Z, q \neq 0 \}$$

For example, ½, ¾, 5/7, 1/3 are rational numbers. The integers 1, 2, 3… are also rational numbers because we write them as 1/1, 2/1, 3/1, … respectively.

Rational numbers can be identified by TWO rules:

1) Rational numbers have finite number of digits, e.g. ½ = 2.5
2) Rational numbers have infinite number of digits after decimal point, but they repeat themselves.

For example, 1/3 = 0.3333333… Here, 3 repeats up to infinity.

Division by zero is not allowed in mathematics.

In mathematics, division by zero is not allowed. We explain it by the following:

Suppose that x is any non-zero real number such that if we divide x by zero, then we get a number y:

$$\frac{x}{0} = y$$

$$x = 0.y$$

$x = 0$ which contradicts to the fact that x is a non-zero number. Hence x cannot be divided by zero.

Suppose that $x = 0$, then $\frac{x}{0} = y$ becomes $\frac{0}{0} = y$.

But $\frac{0}{0}$ is undefined.

NOTE

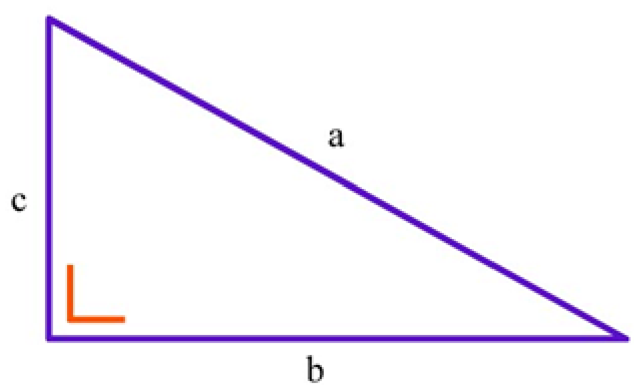
That is why when we take rational number $\frac{p}{q}$, then we put condition to q that q must be non-zero.

PYTHAGORAS

The ancient Greek philosopher and mathematician Pythagoras studied the properties of rational number. He never believed in the existence of Irrational numbers.

Pythagoras (571 BC – 495 BC)

<u>Pythagoras Theorem</u>

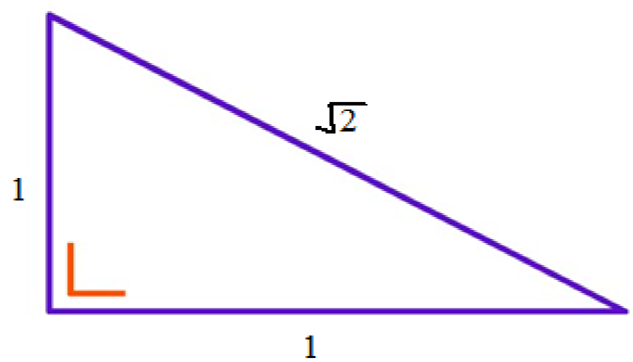


For a right angled triangle,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$$

$$a^2 = b^2 + c^2$$

A student of Pythagoras showed the existence of IRRATIONAL numbers, using Pythagoras theorem.



$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$$

$$= (1)^2 + (1)^2$$

$$(\text{hypotenuse})^2 = 2$$

$$\text{Hypotenuse} = \sqrt{2}$$

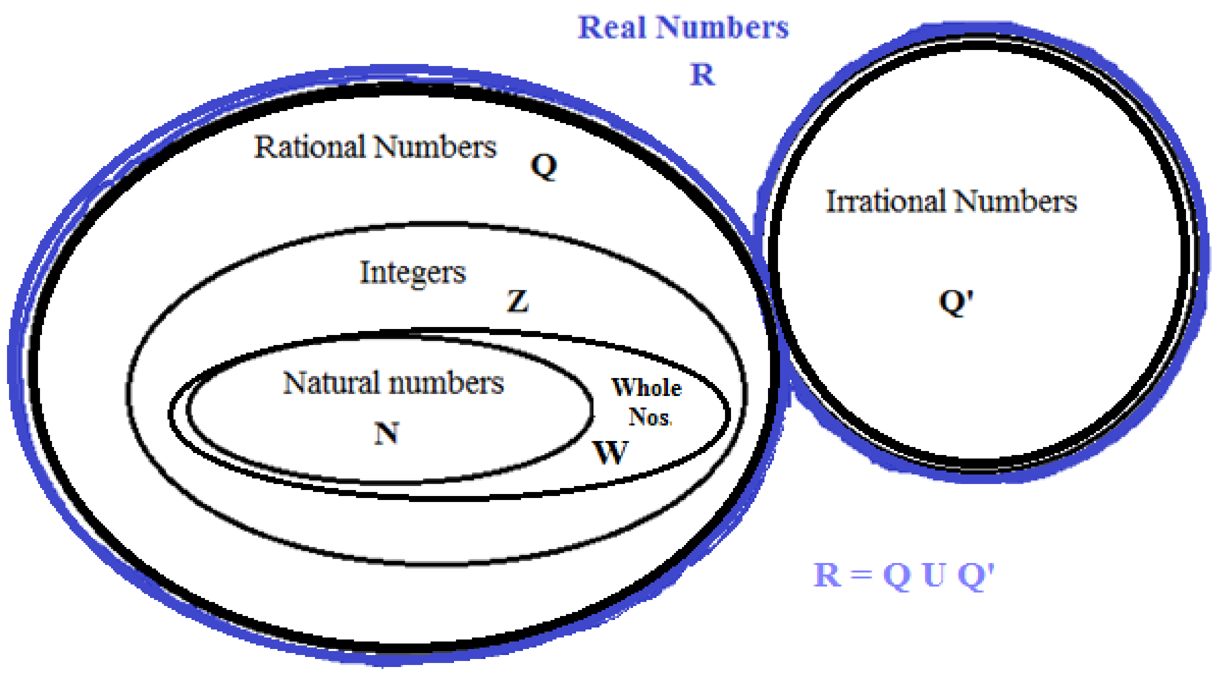
$\sqrt{2}$ is an irrational number.

IRRATIONAL NUMBERS

The irrational numbers are those which cannot be written as ratio $\frac{p}{q}$ of two integers p and q. Since $\sqrt{2} = \frac{\sqrt{2}}{1}$ is not a rational number because $\sqrt{2}$ is not an integer.

An irrational number can be identified with the help of following rules:

- An irrational number has infinite number of digits which do not repeat, e.g. 1.4142135623730950488016 8…
- The square root of a number which is not a perfect square is an irrational number, e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \ldots$ etc.



$$N \subset Z \subset Q \subset R$$

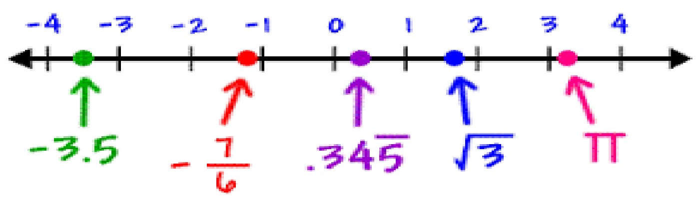
Analytic Geometry

In 1600s, French mathematician Descartes developed the idea of Analytic Geometry.

Analytic Geometry gives ways for describing algebraic formulae by geometric curves and geometric curves by the algebraic formulae or equations.

Coordinate Line

Descartes established a relation between the set of real numbers and the straight line, which is called coordinate line.



Order

Order shows whether a number x is greater another number y or x is smaller than y.

Suppose that 5 is greater than 3, we write it as $5 > 3$.

We can also say it as 3 is smaller than 5, i.e. $3 < 5$.

The bigger number points towards the smaller number, and says, 'I am bigger than you. '

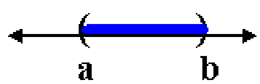
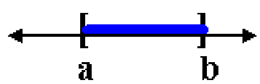
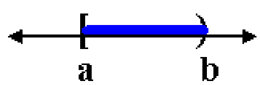
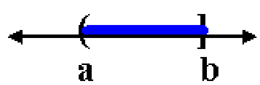
5  3

$5 \longrightarrow 3$ or $5 > 3$

Interval

An interval is a subset of the set of real numbers.

Geometrically, an interval is a line segment between two points a and b.

Picture	Interval Notation	Set Builder Notation	Description
	(a, b)	$\{ x \mid a < x < b \}$	The open interval from a to b. Contains neither endpoint.
	[a, b]	$\{ x \mid a \leq x \leq b \}$	The closed interval from a to b. Contains both endpoints.
	[a, b)	$\{ x \mid a \leq x < b \}$	The interval from a to b Contains a but not b
	(a, b]	$\{ x \mid a < x \leq b \}$	The interval from a to b. Contains b but not a.

Exercise

Show the following sets in interval form. Follow the first example.

1) $\{x \mid 3 < x < 7\} = (3, 7)$
2) $\{x \mid 2 \leq x < 5\} = ?$
3) $\{x \mid 0 < x \leq 9\} = ?$
4) $\{x \mid 11 \leq x \leq 17\} = ?$
5) $\{x \mid -3 \leq x < -17\} = ?$

Solving the inequalities

Example

Solve $3 + 7x \leq 2x - 9$

Solution:

$$3 + 7x \leq 2x - 9$$

Subtract 3 from both sides,

$$-3 + 3 + 7x \leq -3 + 2x - 9$$

$$7x \leq 2x - 12$$

Subtract 2x from both sides,

$$-2x + 7x \leq -2x + 2x - 12$$

$$5x \leq -12$$

Divide 5 on both sides,

$$\frac{5x}{5} \leq \frac{-12}{5}$$

$$x \leq \frac{-12}{5}$$

The solution set is $\left(-\infty, \frac{-12}{5}\right]$

Example

Solve $7 \leq 2 - 5x < 9$

Solution:

$$7 \leq 2 - 5x < 9$$

Subtract 2 from each side,

$$7 - 2 \leq 2 - 5x - 2 < 9 - 2$$

$$5 \leq -5x < 7$$

To make x alone, we divide –5 on each side. When –ve number is multiplied by the inequality, then the inequality signs are changed (reversed).

$$\frac{5}{-5} \geq \frac{-5x}{-5} > \frac{7}{-5}$$

$$-1 \geq x > \frac{-7}{5}$$

$$\frac{-7}{5} < x \leq -1$$

$$\left(\frac{-7}{5}, -1 \right]$$

Example

Solve $(x + 2)(x - 5) > 0$

Solution:

There are two cases:

Case (a): When both $(x + 2)$ and $(x - 5)$ are +ve.

$(x + 2) > 0 \quad \text{and} \ (x - 5) > 0$

$x > -2 \quad \text{and} \quad x > 5$

It means $x > 5$

So the solution is $(5, \infty)$

Case (b): When both $(x + 2)$ and $(x - 5)$ are –ve.

$(x + 2) < 0 \quad \text{and} \ (x - 5) < 0$

$x < -2 \quad \text{and} \quad x < 5$

It means $x < -2$

So the solution is $(-\infty, -2)$

Therefore, the whole solution set is $(-\infty, -2) \cup (5, \infty)$.