MTH101: Solution of Practice Exercise
Lecture No.6: Functions

Q.No.1

Find the natural domain and the range of the given function

$$h(x) = \cos^2(\sqrt{x}).$$

Solution:

As we know that the $\sqrt{x}$ is defined on non-negative real numbers $x \geq 0$. This means that the natural domain of $h(x)$ is the set of positive real numbers.
Therefore, the natural domain of $h(x) = [0, +\infty)$.

As we also know that the range of trigonometric function $\cos x$ is $[-1, 1]$.

The function $\cos^2 \sqrt{x}$ always gives positive real values within the range 0 and 1 both inclusive.

From this we conclude that the range of $h(x) = [0, 1]$.

Q.No.2

Find the domain and range of function f defined by $f(x) = x^2 - 2$.

Solution:

$$\because \quad f(x) = x^2 - 2$$

The domain of this function is the set of all real numbers.
The range is the set of values that $f(x)$ takes as x varies. If x is a real number, x^2 is either positive or zero. Hence we can write the following:

$$x^2 \geq 0,$$

Subtract -2 on both sides to obtain

$$x^2 - 2 \geq -2.$$

The last inequality indicates that $x^2 - 2$ takes all values greater than or equal to -2. The range of function f is the set of all values of $f(x)$ in the interval $[-2, +\infty)$.

Q.No.3

Determine whether $y = \pm\sqrt{x+3}$ is a function or not? Justify your answer.

Solution:

$$\because \quad y = \pm\sqrt{x+3}$$

This is not a function because each value that is assigned to '*x*' gives two values of *y*
So this is not a function. For example, if *x*=1 then

$$y = \pm\sqrt{1+3}\ ,$$
$$y = \pm\sqrt{4},$$
$$y = \pm 2.$$

Q.No.4

Determine whether $y = \dfrac{x+2}{x+3}$ is a function or not? Justify your answer.

Solution:

$$\because \quad y = \frac{x+2}{x+3}$$

This is a function because each value that is assigned to '*x*' gives only one value of *y*
So this is a function. For example if *x*=1 then

$$y = \frac{1+2}{1+3},$$
$$y = \frac{3}{4},$$
$$y = 0.75.$$

Q.No.5

(a) Find the natural domain of the function $f(x) = \dfrac{x^2-16}{x-4}$.

(b) Find the domain of function f defined by $f(x) = \dfrac{-1}{(x+5)}$.

Solution:

(a)

$$\because \quad f(x) = \frac{x^2 - 16}{x - 4},$$

$$\Rightarrow \quad f(x) = \frac{(x+4)(x-4)}{(x-4)},$$

$$= (x+4) \qquad ; \; x \neq 4 \; .$$

This function consists of all real numbers x, except $x = 4$.

(b)

$$\because \quad f(x) = \frac{-1}{(x+5)}$$

This function consists of all real numbers x, except $x = -5$. Since $x = -5$ would make the denominator equal to zero and the division by zero is not allowed in mathematics.

Hence the domain in interval notation is given by $(-\infty, -5) \cup (-5, +\infty)$.