MTH101: Solution of Practice Exercise
Lecture No.5: Distance, Circles, Equations

Solution 1:

The formula to find the distance between any two points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given as

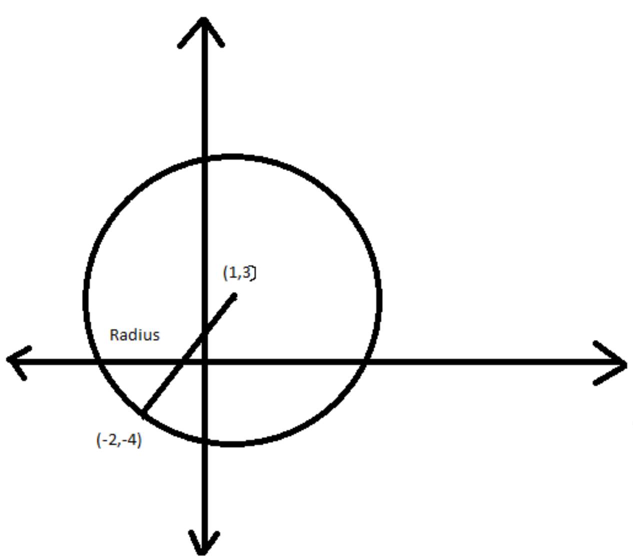
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The given points are $(5,6)$ and $(2,4)$, so the distance between these two points will be

$$\begin{aligned} d &= \sqrt{(2-5)^2 + (4-6)^2} \\ &= \sqrt{(-3)^2 + (-2)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

Solution 2:

It is given that center of the circle is $(1,3)$. We are also given a point on the circle that is $(-2,-4)$ as shown below.



The radius of the circle will be the distance between the points (1,3) and (-2,-4). That is

$$\text{Radius} = d = \sqrt{[1-(-2)]^2 + [3-(-4)^2}$$
$$= \sqrt{(3)^2 + (7)^2}$$
$$= \sqrt{9+49} = \sqrt{58}$$

Solution 3:

The general form of the equation of circle is given as

$$4x^2 + 4y^2 - 16x - 24y + 51 = 0$$

It can be re-written as

$$(4x^2 - 16x) + (4y^2 - 24y) = -51$$

$$(2\,x)^2 - 2(8\,x) + (2\,y)^2 - 2(12\,y) = -51$$

In order to complete the squares on the left hand side, we have to add 16 and 36 on both sides, it will then become

$$(2\,x)^2 - 2(8\,x) + 16 + (2\,y)^2 - 2(12\,y) + 36 = -51 + 16 + 36$$
$$(2\,x)^2 - 2(2\,x)(4) + (4^2) + (2\,y)^2 - 2(2\,y)(6) + (6)^2 = 1$$
$$(2\,x - 4)^2 + (2\,y - 6)^2 = 1$$
$$(x-2)^2 + (y-3)^2 = \left(\frac{1}{4}\right)$$

Comparing it with the standard form of the equation, the center of the circle will be $(2,3)$ and the radius will be $\frac{1}{2}$.

Solution 4:

The general form of the equation of circle is given as

$$2x^2 + 2y^2 + 6x - 8y + 12 = 0$$

It can be re-written as

$$(2x^2 + 6x) + (2y^2 - 8y) = -12$$
$$(x^2 + 3x) + (y^2 - 4y) = -6$$

In order to complete the squares on the left hand side, we have to add $\frac{9}{4}$ and 4 on both sides, it will then become

$$(x^2+3x+\frac{9}{4})+(y^2-4y+4)=-6+\frac{9}{4}+4$$

$$(\mathrm{x}^2+2(\mathrm{x})\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2+(\mathrm{y})^2-2(\mathrm{y})(2)+(2)^2=\frac{1}{4}$$

$$\left(\mathrm{x}+\frac{3}{2}\right)^2+\left(y-2\right)^2=\frac{1}{4}$$

Comparing it with the standard form of the equation, the center of the circle will be $\left(-\frac{3}{2},2\right)$ and radius will be $\frac{1}{2}$.

Solution 5:

The general form of the equation of circle is given as

$$x^2+y^2-4x-6y+8=0$$

This can be re-written as

$$(x^2-4x)+(y^2-6y)=-8$$

In order to complete the squares on the left hand side, we have to add 4 and 9 on both sides, it will then become

$$(x^2-4x+4)+(y^2-6y+9)=-8+4+9$$

$$(\mathrm{x})^2-2(\mathrm{x})(2)+(2)^2+(\mathrm{y})^2-2(\mathrm{y})(3)+(3)^2=5$$

$$(\mathrm{x}-2)^2+(\mathrm{y}-3)^2=5$$

Comparing it with the standard form of the equation, the center of the circle will be (2,3) and the radius will be $\sqrt{5}$.

SOLUTION 6:

$\because\ 3x^2+6x+3y^2+18y-6=0,$

$\Rightarrow 3(x^2+2x+y^2+6y-2)=0, \quad (\because \text{ taking 3 as common})$

$\Rightarrow x^2+2x+y^2+6y-2=0, \quad (\because \text{ dividing by 3 on both sides})$

$\Rightarrow x^2+2x+1+y^2+6y+9=2+9+1,$

$\Rightarrow (x+1)^2+(y+3)^2=12,$

$\Rightarrow (x+1)^2+(y+3)^2=(\sqrt{12})^2,$

$\Rightarrow (x-(-1))^2+(y-(-3))^2=(\sqrt{12})^2,$

$\therefore$ Centre of the circle is (-1,-3) and radius is $\sqrt{12}$.

SOLUTION 7:

$x^2-6x+y^2-8y=0, \quad (\because \text{ rearranging the term})$

$x^2-6x+y^2-8y+(3)^2=(3)^2, \quad (\because \text{ adding } (3)^2 \text{ on both sides})$

$(x^2-6x+9)+y^2-8y=9,$

$(x^2-6x+9)+y^2-8y+(4)^2=9+(4)^2, \quad (\because \text{ adding } (4)^2 \text{ on both sides})$

$(x^2-6x+9)+(y^2-8y+16)=9+16,$

$(x-3)^2+(y-4)^2=9+16,$

$(x-3)^2+(y-4)^2=(\sqrt{25})^2, \_\_\_\_\_\_ \text{ eq.(1)}$

$\because\ (x-x_0)^2+(y-y_0)^2=r^2. \quad \_\_\_\_\_\_ \text{ eq.(2)}$

The eq.(1) is now in the standard form of eq.(2). This equation represents a circle with the center at (3, 4) and with a radius equal to $\sqrt{25}$.

SOLUTION 8:

The standard form of equation of circle is

$(x-h)^2+(y-k)^2=r^2,$

Here $h=3,\ k=-2,\ r=4,$

$(x-3)^2+(y-(-2))^2=4^2,$

$x^2-6x+9+y^2+4+4y=16,$

$x^2+y^2-6x+4y=16-9-4,$

$x^2+y^2-6x+4y=3.$

SOLUTION 9:

The distance formula between two points (x_1, y_1) and (x_2, y_2) in a coordinate plane is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \ ,$$
$$d = \sqrt{(8-2)^2 + (6-4)^2} \ ,$$
$$= \sqrt{(6)^2 + (2)^2} \ ,$$
$$= \sqrt{36+4} \ ,$$
$$= \sqrt{40} \ ,$$
$$= 2\sqrt{10}.$$

<u>SOLUTION 10:</u>

The radius is the distance between the center and any point on the circle, so find the distance:

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \ ,$$
$$r = \sqrt{(3-(-1))^2 + (-2-(-3))^2} \quad ,$$
$$= \sqrt{(3+1)^2 + (-2+3)^2} \quad ,$$
$$= \sqrt{(4)^2 + (1)^2} \quad ,$$
$$= \sqrt{16+1} \quad ,$$
$$= \sqrt{17} \ ,$$
$$\approx 4.123.$$

Then the radius is $\sqrt{17}$, or about 4.123, rounded to three decimal places.