MTH101: Solution of Practice Exercise
Lecture No.20: Derivatives of Logarithmic and Exponential Functions

Q.No.1

Differentiate: $y=(5-x)^{\sqrt{x}}$.

Answer: $\dfrac{dy}{dx}=\left(\dfrac{\ln(5-x)}{2\sqrt{x}}-\dfrac{\sqrt{x}}{(5-x)}\right).(5-x)^{\sqrt{x}}$

Solution:

$$\because \quad y=(5-x)^{\sqrt{x}} \ ,$$

taking log on both sides ,

$$\Rightarrow \ln y=\sqrt{x}\,\ln(5-x) \qquad \left(\because \ln m^n = n\ln m\right),$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{1}{2\sqrt{x}}.\ln(5-x)+\frac{1}{(5-x)}(-1).\sqrt{x} \ ,$$

$$\Rightarrow \frac{dy}{dx}=\left(\frac{\ln(5-x)}{2\sqrt{x}}-\frac{\sqrt{x}}{(5-x)}\right).y \ ,$$

$$\Rightarrow \frac{dy}{dx}=\left(\frac{\ln(5-x)}{2\sqrt{x}}-\frac{\sqrt{x}}{(5-x)}\right).(5-x)^{\sqrt{x}} \ .$$

Q.No.2

Differentiate $y=(\cos x)^{8x}$ with respect to 'x'.

Answer: $\dfrac{dy}{dx}=\left(8\,\ln(\cos x)-\dfrac{8x\sin x}{\cos x}\right)(\cos x)^{8x}$

Solution:

$\because \quad y=(\cos x)^{8x}$,

taking log on both sides,

$\Rightarrow \ln y=(8x)\ln(\cos x)$, $\quad\quad (\because \ln m^n = n\ln m)$,

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=8.\ln(\cos x)+\frac{1}{(\cos x)}.(-\sin x).(8x)$, $\quad\quad \left(\because \frac{d}{dx}(\ln x)=\frac{1}{x}\ ;\ \frac{d}{dx}(\cos x)=-\sin x\right)$,

$\Rightarrow \frac{dy}{dx}=\left(8\ \ln(\cos x)-\frac{8x\sin x}{\cos x}\right).y$,

$\Rightarrow \frac{dy}{dx}=\left(8\ \ln(\cos x)-\frac{8x\sin x}{\cos x}\right)(\cos x)^{8x}$.

Q.No.3

Differentiate $y=x^{\sin 5x}$ with respect to 'x'.

Answer: $\frac{dy}{dx}=\left(5(\cos 5x).\ln(x)+\frac{\sin 5x}{x}\right)\left(x^{\sin 5x}\right)$

Solution:

$\because \quad y=x^{\sin 5x}$,

Taking log on both sides ,

$\Rightarrow \ln y=(\sin 5x)\ln(x)$, $\quad\quad (\because \ln m^n = n\ln m)$,

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=5(\cos 5x).\ln(x)+\frac{1}{x}.(\sin 5x)$, $\quad\quad \left(\because \frac{d}{dx}(\ln x)=\frac{1}{x}\ ;\ \frac{d}{dx}(\sin x)=\cos x\right)$,

$\Rightarrow \frac{dy}{dx}=\left(5(\cos 5x).\ln(x)+\frac{\sin 5x}{x}\right).y$,

$\Rightarrow \frac{dy}{dx}=\left(5(\cos 5x).\ln(x)+\frac{\sin 5x}{x}\right)\left(x^{\sin 5x}\right)$.

Q.No.4

Differentiate $y=x\ e^{3x+4}$.

Answer: $\frac{dy}{dx}=e^{3x+4}+3xe^{3x+4}$

Solution:

$$\because \quad y = x\, e^{3x+4} ,$$

$$\Rightarrow \frac{dy}{dx} = e^{3x+4} + xe^{3x+4}\frac{d}{dx}(3x+4) ,$$

$$\Rightarrow \frac{dy}{dx} = e^{3x+4} + 3xe^{3x+4} .$$

Q.No.5

Find the derivative of the function $y = \ln(2 + x^5)$ with respect to 'x'.

Answer: $\frac{dy}{dx} = \frac{5x^4}{(2+x^5)}$

Solution:

$$y = \ln(2 + x^5) ,$$

now taking the derivative of the function on both sides ,

$$\frac{dy}{dx} = \frac{d}{dx}\{\ln(2+x^5)\} ,$$

$$\frac{dy}{dx} = \frac{1}{(2+x^5)}\frac{d}{dx}(2+x^5) ,$$

$$\frac{dy}{dx} = \frac{1}{(2+x^5)}(0+5x^4) ,$$

$$\frac{dy}{dx} = \frac{5x^4}{(2+x^5)} .$$