MTH101: Solution of Practice Exercise
Lecture No.17: Derivatives of Trigonometric Functions

Q.No.1

Find $\frac{dy}{dx}$ if $y = x^3 \cot x - \frac{3}{x^3}$.

Answer: $3x^2 \cot x - x^3 cosec^2 x + \frac{9}{x^4}$

Solution:

Given $y = x^3 \cot x - \frac{3}{x^3}$,

$\frac{dy}{dx} = \cot x \frac{d}{dx}(x^3) + x^3 \frac{d}{dx}(\cot x) - \frac{d}{dx}\left(\frac{3}{x^3}\right)$,

$= \cot x\,(3x^2) + x^3(-cosec^2 x) - 3\frac{d}{dx}\left(\frac{1}{x^3}\right)$,

$= 3x^2 \cot x - x^3 cosec^2 x + \frac{9}{x^4}$ (Answer).

Q.No.2

Find $\frac{dy}{dx}$ if $y = x^4 \sin x$ at $x = \pi$.

Answer: $-\pi^4$

Solution:

$\because \frac{d}{dx}(f.g) = f.\frac{d}{dx}(g) + g.\frac{d}{dx}(f)$,

$y = x^4 \sin x$ at $x = \pi$,

$\frac{d}{dx} = \sin x \frac{d}{dx}(x^4) + x^4 \frac{d}{dx}(\sin x)$,

$= \sin x\ (4x^3) + x^4(\cos x)$,

$= 4x^3 \sin x + x^4 \cos x$,

$= 4\pi^3 \sin \pi + \pi^4 \cos \pi$, at $x = \pi$,

$= 4\pi^3(0) + \pi^4(-1)$,

$= -\pi^4$ (Answer).

Q.No.3

Find $f'(t)$ if $f(t) = \frac{2-8t+t^2}{sint}$.

Answer: $\frac{[(2t-8)(\sin t)]-[(t^2-8t+2)(\cos t)]}{\sin^2 t}$

Solution:

$\because \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).\frac{d}{dx}(f(x)) - f(x).\frac{d}{dx}(g(x))}{[g(x)]^2}$,

$f(t) = \frac{2-8t+t^2}{sint}$,

$f'(t) = \frac{[(\sin t)(-8+2t)]-[(2-8t+t^2)(\cos t)]}{(\sin t)^2}$,

$= \frac{[(2t-8)(\sin t)]-[(t^2-8t+2)(\cos t)]}{\sin^2 t}$ (Answer).

Q.No.4

Find $f'(y)$ if $(y) = \frac{\sin y + 3\tan y}{y^3 - 2}$.

Answer: $\frac{[(y^3-2)(\cos y + 3\sec^2 y)] - [(\sin y + 3\tan y) + (3\,y^2)]}{y^6 - 4y^3 + 4}$

Solution:

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).\frac{d}{dx}(f(x)) - f(x).\frac{d}{dx}(g(x))}{[g(x)]^2}$,

$f(y) = \frac{\sin y + 3\tan y}{y^3 - 2}$.

$f'(y) = \frac{[(y^3-2)(\cos y + 3\sec^2 y)] - [(\sin y + 3\tan y) + (3\,y^2)]}{(y^3-2)^2}$,

$= \frac{[(y^3-2)(\cos y + 3\sec^2 y)] - [(\sin y + 3\tan y) + (3\,y^2)]}{y^6 - 4y^3 + 4}$ (Answer).

Q.No.5

(a) Find $\frac{dy}{dx}$ if $y = (5x^2 + 3x + 3)(\sin x)$.

(b) Find $f'(t)$ if $(t) = 5t \sin t$.

Answer: **(a)** $(5\,x^2 + 3x + 3)(\cos x) + \sin x\,.(10x + 3)$

(b) $5\,t\cos t + 5\sin t$

Solution:

(a) $\because \frac{d}{dx}(f.g) = f.\frac{d}{dx}(g) + g.\frac{d}{dx}(f)$,

$y = (5x^2 + 3x + 3)(\sin x)$,

$\frac{d}{dx}[(5\,x^2 + 3x + 3)(\sin x)] = (5\,x^2 + 3x + 3)(\cos x) + \sin x\,(10x + 3)$ (Answer).

(b) $\because \frac{d}{dx}(f.g) = f.\frac{d}{dx}(g) + g.\frac{d}{dx}(f)$,

$f(t) = 5t \sin t$,

$\frac{d}{dx}(5\,t \sin t) = 5\,t\cos t + (\sin t)(5)$,

$= 5\,t\cos t + 5\sin t$ (Answer).