Lecture 8

Graphs of Functions

- Represent Functions by graphs
- Visualize behavior of Functions through graphs
- How to use graphs of simple functions to create graphs of complicated functions

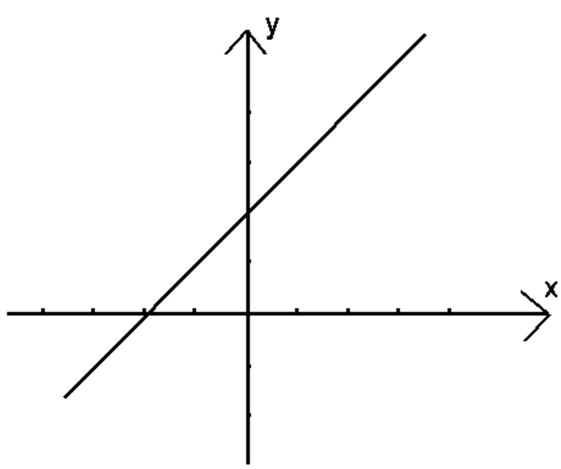
Definition of Graph of a function

- We saw earlier the relationship between graph and its equation
- A graph of an equation is just the points on the xy- plane that satisfy the equation
- Similarly, the graph of a Function f in the xy-plane is the GRAPH of the equation y = f (x)

Example

Sketch the graph of $f(x) = x+2$

By definition, the graph of f is the graph of $y = x+2$. This is just a line with y-intercept 2 and slope 1. We saw how to plot lines in a previous lecture



graph of f(x) = x + 2

Example

Sketch the graph of $f(x) = |x|$

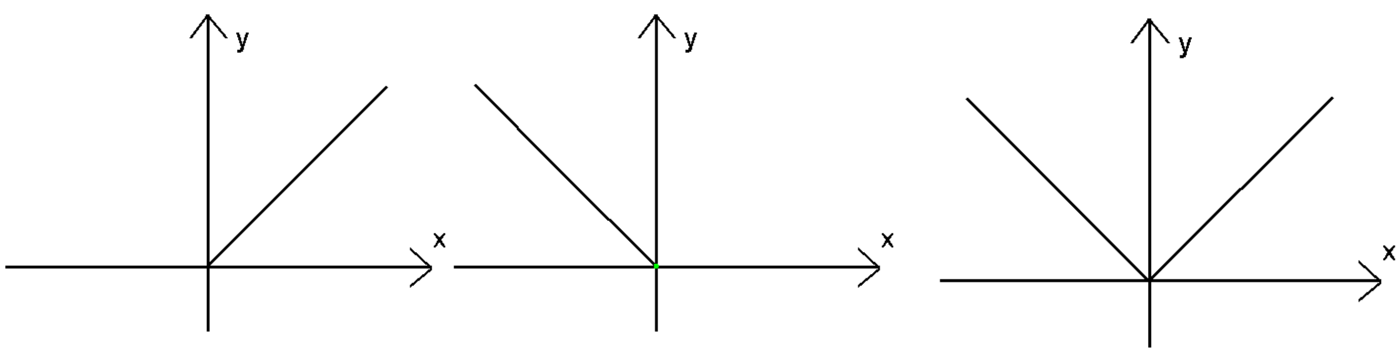
The graph will be that of $y = |x|$

Remember that absolute value is defined as

$$y = |x| = \begin{cases} x & if \;\; x \geq 0 \\ -x & if \;\; x < 0 \end{cases}$$

Remember that absolute value function is PIECEWISE defined. The top part is the function y = f(x) = x, the bottom is the function y = f(x) = -x

y = x is just a straight line through the origin with slope 1, but is only defined for x > = 0.
y = -x is a straight line through the origin with slope –1 but defined only for x < 0.
Here is the GRAPH



graph of f(x) = x   graph of f(x) = -x   graph of f(x) = |x|

EXAMPLE

$$t(x) = \frac{x^2 - 4}{x - 2}$$

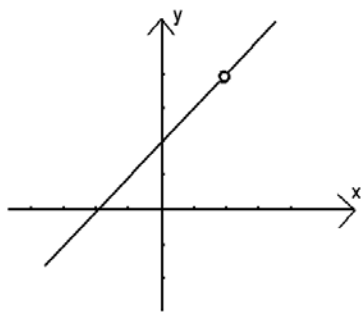
This is the same as $t(x) = x + 2 \quad x \neq 2$

Graph will be of $y = x + 2, \quad x \neq 2$

Remember from Lecture 6??

t(x) is the same as in example 1, except that 2 is not part of the domain, which means there is no y value corresponding to x = 2. So there is a HOLE in the graph.

Picture



graph of y = x + 2 x ≠ 2

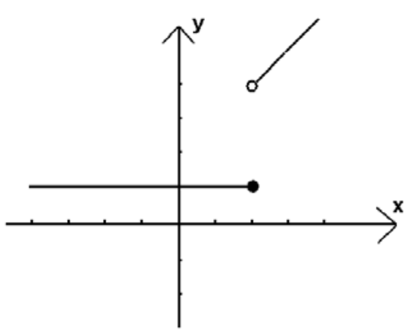
EXAMPLE

$$g(x) = \begin{cases} 1 & \text{if } x \leq 2 \\ x+2 & \text{if } x > 2 \end{cases}$$

Graph will be of

$$y = \begin{cases} 1 & \text{if } x \leq 2 \\ x+2 & \text{if } x > 2 \end{cases}$$

For x is less than or equal to 2, the graph is just at y = 1. This is a straight line with slope 0. For x is greater than 2, the graph is the line x+2



Graphing functions by Translations

Suppose the graph of *f(x)* is known.
Then we can find the graphs of $y = f(x)+c,\ \ y = f(x)-c,\ \ y = f(x+c),\ \ y = f(x-c)$.

c is any POSITIVE constant.

- If a positive constant *c* is added to *f (x)* ,the geometric effect is the translation of the graph of *y=f (x)* UP by *c* units.
- If a positive constant *c* is subtracted from *f (x)* the geometric effect is the translation of the graph of *y=f (x)* DOWN by *c* units.
- If a positive constant *c* is added to the independent variable *x* of *f (x)* ,the geometric effect is the translation of the graph of *y=f (x)* LEFT by *c* units.
- If a positive constant *c* is subtracted from the independent variable *x* of *f (x),* the geometric effect is the translation of the graph of *y=f (x)* RIGHT by *c* units.

Here is a table summarizing what we just talked about in terms of translations of function

y = f(x) + c	graph of f(x) translates *UP* by c units
y = f(x) - c	graph of f(x) translates *DOWN* by c units
y = f(x + c)	graph of f(x) translates *LEFT* by c units
y = f(x - c)	graph of f(x) translates *RIGHT* by c units

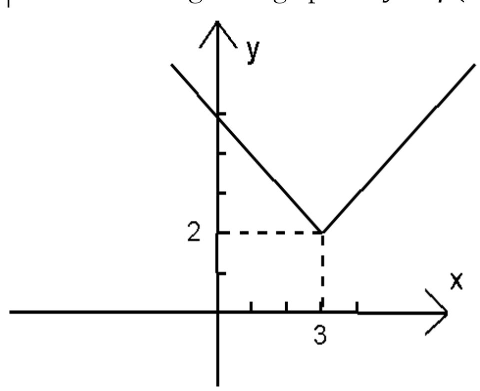
Example

Sketch the graph of $y = f(x) = |x-3| + 2$

This graph can be obtained by two translations

Translate the graph of $y = |x|$ 3 units to the RIGHT to get the graph of $y = |x-3|$.

Translate the graph of $y = |x-3|$ 2 units UP to get the graph of $y = f(x) = |x-3| + 2$.



graph of f(x) = |x - 3| + 2

Example

Sketch graph of $y = x^2 - 4x + 5$

Complete the square

Divide the co-efficient of x by 2

Square this result and add to the both sides of your equation

$y + 4 = (x^2 - 4x + 5) + 4$

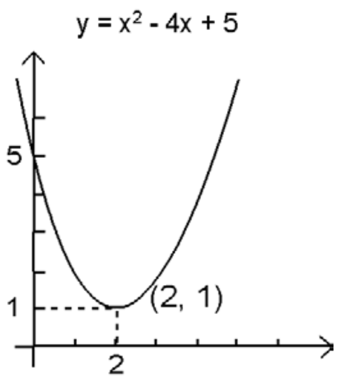
$y = (x^2 - 4x + 4) + 5 - 4$

$y = (x-2)^2 + 1$

This equation is the same as the original

Graph

$y = x^2$



Shift it RIGHT by 2 units to get graph of $y = (x-2)^2$

Shift this UP by 1 Unit to get $y = (x-2)^2 + 1$

Reflections

From Lecture 3

- (-x, y) is the reflection of (x, y) about the y-axis
- (x, -y) is the reflection of (x, y) about the x-axis

Graphs of y = f(x) and y = f(-x) are reflections of one another about the y-axis.
Graphs of y = f(x) and y = -f(x) are reflections of one another about the x-axis.

Example

Sketch the graph of $y = \sqrt[3]{2-x}$

We can always plot points by choosing x-values and getting the corresponding y-values. Better if we get the graph by REFELCTION and TRANSLATION

Here is HOW

- First graph $y = \sqrt[3]{x}$
- Reflect it about the y-axis to get graph of $y = \sqrt[3]{-x}$

  Remember that negative numbers HAVE cube roots
- Translate this graph RIGHT by 2 units to get graph of

  $y = \sqrt[3]{2-x} = \sqrt[3]{-x+2} = \sqrt[3]{-(x-2)}$ scaling.

If *f (x)* is MULTIPLIED by a POSITIVE constant *c*, then the following geometric effects take place

- The graph of *f (x)* is COMPRESSED vertically if $0 < c < 1$
- The graph of *f (x)* is STRETCHED vertically if $c > 1$
- This is called VERTICAL Scaling by a factor of *c*.
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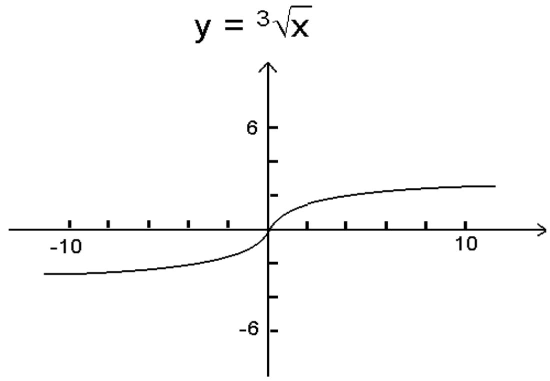

Example

$$y = 2\sin(x)$$
$$y = \sin(x)$$
$$y = \frac{1}{2}\sin(x)$$

$c = 2$ and $c=1/2$ is used and here are the corresponding graphs with appropriate VERTICAL SCALINGS

Vertical Line Test

- So far we have started with a function equation, and drawn its graph
- What if we have a graph given. Must it be the graph of a function??
- Not every curve or graph in the xy-plane is that of a function

Example

Here is a graph which is not the graph of a function. Figure 2.3.12

Its not a graph because if you draw a VERTICAL line through the point x = a, then the line crosses the graph in two points with y values y = b, y = c.

This gives you two points on the graph namely

(a,b) and (a,c)

But this cannot be a function by the definition of a function.

VERTICAL LINE TEST

A graph in the plane is the graph of a function if and only if NO VERTICAL line intersects the graph more than once.

Example

$x^2 + y^2 = 25$

The graph of this equation is a CIRLCE. Various vertical lines cross the graph in more than 2 places. So the graph is not that of a function which means that equations of Circles are not functions x as a function of y. A given graph can be a function with y independent and x dependent. That is why, it could be the graph of an equation like x = g(y).

This would happen if the graph passes the HORIZONTAL LINE test.

This is so because for each y, there can be only one x by definition of FUNCTION.

Also,

$y = x^2$ Gives

$g(y) = x = \pm\sqrt{y}$

SO for each x, two y's and its not a function in y as clear from the graph of the function