

MTH101 Solution: Practice Questions
Lecture No.8: Graphs of Functions
Lecture No.9: Limits

Choose the correct option for the following questions:

- 1) If a vertical line intersects the graph of the equation $y = f(x)$ at two points, then which of the following is true?
- I. It represents a function.
 - II. It represents a parabola.
 - III. It represents a straight line.
 - IV. **It does not represent a function.** Correct option
- 2) Which of the following is the reflection of the graph of $y = f(x)$ about y -axis?
- I. $y = -f(x)$
 - II. **$y = f(-x)$** Correct option
 - III. $-y = -f(x)$
 - IV. $-y = f(-x)$
- 3) Given the graph of a function $y = f(x)$ and a constant c , the graph of $y = f(x) + c$ can be obtained by _____.
- I. **Translating the graph of $y = f(x)$ up by c units.** Correct option
 - II. Translating the graph of $y = f(x)$ down by c units.
 - III. Translating the graph of $y = f(x)$ right by c units.
 - IV. Translating the graph of $y = f(x)$ left by c units.
- 4) Given the graph of a function $y = f(x)$ and a constant c , the graph of $y = f(x - c)$ can be obtained by _____.
- I. Translating the graph of $y = f(x)$ up by c units.
 - II. Translating the graph of $y = f(x)$ down by c units.
 - III. **Translating the graph of $y = f(x)$ right by c units.** Correct option
 - IV. Translating the graph of $y = f(x)$ left by c units.
- 5) Which of the following is the reflection of the graph of $y = f(x)$ about x -axis?
- I. **$y = -f(x)$** Correct option
 - II. $y = f(-x)$
 - III. $-y = -f(x)$
 - IV. $-y = f(-x)$

Q.No.6: If $\lim_{x \rightarrow 8^-} h(x) = 18 + c$ and $\lim_{x \rightarrow 8^+} h(x) = 7$ then find the value of 'c' so that $\lim_{x \rightarrow 8} h(x)$ exists.

Answer: $c = -11$

Solution: For the existence of $\lim_{x \rightarrow 8} h(x)$ we must have

$$\lim_{x \rightarrow 8^-} h(x) = \lim_{x \rightarrow 8^+} h(x),$$

By placing the values we get

$$\begin{aligned} 18 + c &= 7, \\ \Rightarrow c &= 7 - 18 = -11. \end{aligned}$$

Q.No.7: Find the limit by using the definition of absolute value $\lim_{x \rightarrow 0^+} \frac{x}{|2x|}$.

Answer: $\frac{1}{2}$

Solution:

$$\therefore \lim_{x \rightarrow 0^+} \frac{x}{|2x|},$$

$$\text{where } |2x| = \begin{cases} 2x & x \geq 0, \\ -2x & x < 0. \end{cases}$$

So $|2x| \rightarrow 2x$ as $x \rightarrow 0^+$.

$$\therefore \lim_{x \rightarrow 0^+} \frac{x}{|2x|} = \lim_{x \rightarrow 0^+} \frac{x}{2x} = \lim_{x \rightarrow 0^+} \frac{1}{2} = \frac{1}{2}.$$

Q.No.8: Find the limit by using the definition of absolute value $\lim_{x \rightarrow 0^-} \frac{|x+5|}{x+5}$.

Answer: -1

Solution:

$$\therefore \lim_{x \rightarrow 0^-} \frac{|x+5|}{x+5}$$

$$\text{where } |x+5| = \begin{cases} x+5 & (x+5) \geq 0, \\ -(x+5) & (x+5) < 0. \end{cases}$$

$$\therefore \lim_{x \rightarrow 0^-} \frac{|x+5|}{x+5} = \lim_{x \rightarrow 0^-} \frac{-(x+5)}{x+5} = \lim_{x \rightarrow 0^-} (-1) = -1.$$

Q.No.9: Evaluate: $\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 1}{x^3 + 2x^2 - 5x + 3}$.

Answer: 0

Solution:

$$\begin{aligned}
\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 1}{x^3 + 2x^2 - 5x + 3} &= \lim_{x \rightarrow \infty} \frac{1 - \frac{3}{x} + \frac{1}{x^3}}{1 + \frac{2}{x} - \frac{5}{x^2} + \frac{3}{x^3}}, \quad (\because \text{taking } x^3 \text{ as common}) \\
&= \frac{1 - \frac{3}{\infty} + \frac{1}{\infty^3}}{1 + \frac{2}{\infty} - \frac{5}{\infty^2} + \frac{3}{\infty^3}}, \quad (\because \text{on applying limit}) \\
&= \frac{0}{1}, \quad (\because \text{any number divided by infinity is zero}) \\
&= 0. \quad \left(\because \frac{0}{1} = 0 \right)
\end{aligned}$$

Q.No.10: Evaluate: $\lim_{z \rightarrow \infty} \frac{z^3 + 2z^2 - 5z + 3}{z^2 - 3z + 1}$.

Answer: ∞

Solution:

$$\begin{aligned}
\lim_{z \rightarrow \infty} \frac{z^3 + 2z^2 - 5z + 3}{z^2 - 3z + 1} &= \lim_{z \rightarrow \infty} \frac{1 + \frac{2}{z} - \frac{5}{z^2} + \frac{3}{z^3}}{\frac{1}{z} - \frac{3}{z^2} + \frac{1}{z^3}}, \quad (\because \text{taking } z^3 \text{ as common}) \\
&= \frac{1 + \frac{2}{\infty} - \frac{5}{\infty^2} + \frac{3}{\infty^3}}{\frac{1}{\infty} - \frac{3}{\infty^2} + \frac{1}{\infty^3}}, \quad (\because \text{on applying limit}) \\
&= \frac{1}{0}, \quad (\because \text{any number divided by infinity is zero}) \\
&= \infty. \quad \left(\because \frac{1}{0} = \infty \right)
\end{aligned}$$