

MTH101 Solution: Practice Questions
Lecture No.7: Operations on Functions

Q.No.1: Consider the functions $f(x) = (x-2)^3$ and $g(x) = \frac{1}{x^2}$. Find the composite function $(f \circ g)(x)$ and also find the domain of this composite function.

Solution:

Domain of $f(x) = -\infty < x < \infty = (-\infty, +\infty)$.

Domain of $g(x) = x < 0$ or $x > 0 = (-\infty, 0) \cup (0, +\infty)$.

$$f \circ g(x) = f(g(x)),$$

$$= f\left(\frac{1}{x^2}\right),$$

$$= \left(\frac{1}{x^2} - 2\right)^3.$$

The domain $f \circ g$ consists of the numbers x in the domain of g such that $g(x)$ lies in the domain of f .
 \therefore Domain of $f \circ g(x) = (-\infty, 0) \cup (0, +\infty)$.

Q.No.2: Let $f(x) = x+1$ and $g(x) = x-2$. Find $(f+g)(2)$.

Solution: From the definition,

$$(f+g)(x) = f(x) + g(x),$$

$$= x+1 + x-2,$$

$$= 2x-1.$$

Hence, if we put $x=2$, we get

$$(f+g)(2) = 2(2)-1 = 3.$$

Q.No.3: Let $f(x) = x^2 + 5$ and $g(x) = 2\sqrt{x}$. Find $(g \circ f)(x)$. Also find the domain of $(g \circ f)(x)$.

Solution:

By definition,

$$(g \circ f)(x) = g(f(x)),$$

$$= g(x^2 + 5),$$

$$= 2\sqrt{x^2 + 5}.$$

Domain of $f(x) = -\infty < x < \infty = (-\infty, +\infty)$.

Domain of $g(x) = x \geq 0 = [0, +\infty)$.

The domain of **gof** is the set of numbers x in the domain of f such that $f(x)$ lies in the domain of g .

Therefore, the domain of $g(f(x)) = (-\infty, +\infty)$.

Q.No.4: Given $f(x) = \frac{3}{x-2}$, and $g(x) = \sqrt{\frac{1}{x}}$. Find the domain of these functions. Also find the intersection of their domains.

Solution:

Here $f(x) = \frac{3}{x-2}$, so

domain of $f(x) = x < 2$ or $x > 2 = (-\infty, 2) \cup (2, +\infty)$.

Now consider $g(x) = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}$.

Domain of $g(x) = x > 0 = (0, +\infty)$.

Also, intersection of domains:

domain of $f(x) \cap$ domain of $g(x) = (0, 2) \cup (2, +\infty)$.

Q.No.5: Given $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x-2}$, find $(f-g)(3)$.

Solution:

$(f-g)(x) = f(x) - g(x)$,

$$= \frac{1}{x^2} - \frac{2}{x-2},$$

$$(f-g)(3) = \frac{1}{9} - \frac{2}{1} = \frac{1-18}{9} = \frac{-17}{9}.$$