Symbolab

Solution

$$\int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}}dx = 3+3\cdot\ 2^{\frac{1}{3}} \quad (\text{Decimal:} \quad 6.77976\ldots)$$

Steps

$$\int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}}dx$$

Adjust integral based on undefined points

There is an undefined point within the boundaries at: 2

If exist b, $a<b<c, f(b)=$ undefined, $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

$$= \int_1^2 \frac{1}{(x-2)^{\frac{2}{3}}}dx + \int_2^4 \frac{1}{(x-2)^{\frac{2}{3}}}dx$$

$$\int_1^2 \frac{1}{(x-2)^{\frac{2}{3}}}dx = 3$$

Hide Steps

$$\int_1^2 \frac{1}{(x-2)^{\frac{2}{3}}}dx$$

Compute the indefinite integral: $\int \frac{1}{(x-2)^{\frac{2}{3}}}dx = 3(x-2)^{\frac{1}{3}} + C$

Hide Steps

$$\int \frac{1}{(x-2)^{\frac{2}{3}}}dx$$

Apply u−substitution: $u=x-2$

Show Steps

$$= \int \frac{1}{u^{\frac{2}{3}}}du$$

$$\frac{1}{u^{\frac{2}{3}}} = u^{-\frac{2}{3}}$$

$$= \int u^{-\frac{2}{3}}du$$

Apply the Power Rule: $\int x^a dx = \frac{x^{a+1}}{a+1}, \quad a \neq -1$

$$=\frac{u^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}$$

Substitute back $u=x-2$

$$=\frac{(x-2)^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}$$

Simplify $\frac{(x-2)^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}$: $3(x-2)^{\frac{1}{3}}$

Hide Steps

$$\frac{(x-2)^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}$$

Join $-\frac{2}{3}+1$: $\frac{1}{3}$

Hide Steps

$$-\frac{2}{3}+1$$

Convert element to fraction: $1=\frac{1\cdot 3}{3}$

$$=-\frac{2}{3}+\frac{1\cdot 3}{3}$$

Since the denominators are equal, combine the fractions: $\frac{a}{c}\pm\frac{b}{c}=\frac{a\pm b}{c}$

$$=\frac{-2+1\cdot 3}{3}$$

$-2+1\cdot 3=1$

Hide Steps

$$-2+1\cdot 3$$

Multiply the numbers: $1\cdot 3=3$

$$=-2+3$$

Add/Subtract the numbers: $-2+3=1$

$$=1$$

$$=\frac{1}{3}$$

$$=\frac{(x-2)^{-\frac{2}{3}+1}}{\frac{1}{3}}$$

$(x-2)^{-\frac{2}{3}+1}=(x-2)^{\frac{1}{3}}$

Hide Steps

$(x-2)^{-\frac{2}{3}+1}$

Join $-\frac{2}{3}+1$: $\frac{1}{3}$

Hide Steps

$-\frac{2}{3}+1$

Convert element to fraction: $1=\frac{1\cdot 3}{3}$

$=-\frac{2}{3}+\frac{1\cdot 3}{3}$

Since the denominators are equal, combine the fractions: $\frac{a}{c}\pm\frac{b}{c}=\frac{a\pm b}{c}$

$=\frac{-2+1\cdot 3}{3}$

$-2+1\cdot 3=1$

Hide Steps

$-2+1\cdot 3$

Multiply the numbers: $1\cdot 3=3$

$=-2+3$

Add/Subtract the numbers: $-2+3=1$

$=1$

$=\frac{1}{3}$

$=(x-2)^{\frac{1}{3}}$

$=\frac{(x-2)^{\frac{1}{3}}}{\frac{1}{3}}$

Apply the fraction rule: $\frac{a}{\frac{b}{c}}=\frac{a\cdot c}{b}$

$=\frac{(x-2)^{\frac{1}{3}}\cdot 3}{1}$

Apply rule $\frac{a}{1}=a$

$=3(x-2)^{\frac{1}{3}}$

$=3(x-2)^{\frac{1}{3}}$

Add a constant to the solution

$=3(x-2)^{\frac{1}{3}}+C$

Compute the boundaries: $\int_1^2 \frac{1}{(x-2)^{\frac{2}{3}}}dx=0-3(-1)^{\frac{1}{3}}$

Hide Steps

$\int_a^b f(x)dx=F(b)-F(a)=\lim_{x\to b-}(F(x))-\lim_{x\to a+}(F(x))$

$\lim_{x\to 1+}\left(3(x-2)^{\frac{1}{3}}\right)=3(-1)^{\frac{1}{3}}$

Hide Steps

$\lim_{x\to 1+}\left(3(x-2)^{\frac{1}{3}}\right)$

Plug in the value $x=1$

$=3(1-2)^{\frac{1}{3}}$

Simplify

$=3(-1)^{\frac{1}{3}}$

$\lim_{x\to 2-}\left(3(x-2)^{\frac{1}{3}}\right)=0$

Hide Steps

$\lim_{x\to 2-}\left(3(x-2)^{\frac{1}{3}}\right)$

Plug in the value $x=2$

$=3(2-2)^{\frac{1}{3}}$

Simplify

$=0$

$=0-3(-1)^{\frac{1}{3}}$

$=0-3(-1)^{\frac{1}{3}}$

Simplify

$=3$

$\int_2^4 \frac{1}{(x-2)^{\frac{2}{3}}}dx=3\cdot\ 2^{\frac{1}{3}}$

Hide Steps

$\int_2^4 \frac{1}{(x-2)^{\frac{2}{3}}}dx$

Compute the indefinite integral: $\int \frac{1}{(x-2)^{\frac{2}{3}}}dx = 3(x-2)^{\frac{1}{3}} + C$

Hide Steps

$\int \frac{1}{(x-2)^{\frac{2}{3}}}dx$

Apply u−substitution: $u = x - 2$

Show Steps

$= \int \frac{1}{u^{\frac{2}{3}}}du$

$\frac{1}{u^{\frac{2}{3}}} = u^{-\frac{2}{3}}$

$= \int u^{-\frac{2}{3}}du$

Apply the Power Rule: $\int x^a dx = \frac{x^{a+1}}{a+1}, \quad a \neq -1$

$= \frac{u^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}$

Substitute back $u = x - 2$

$= \frac{(x-2)^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}$

Simplify $\frac{(x-2)^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}$: $3(x-2)^{\frac{1}{3}}$

Hide Steps

$\frac{(x-2)^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}$

Join $-\frac{2}{3}+1$: $\frac{1}{3}$

Hide Steps

$-\frac{2}{3}+1$

Convert element to fraction: $1 = \frac{1 \cdot 3}{3}$

$= -\frac{2}{3} + \frac{1 \cdot 3}{3}$

Since the denominators are equal, combine the fractions: $\frac{a}{c}\pm\frac{b}{c}=\frac{a\pm b}{c}$

$=\frac{-2+1\cdot 3}{3}$

$-2+1\cdot 3=1$ *Hide Steps*

$-2+1\cdot 3$

Multiply the numbers: $1\cdot 3=3$

$=-2+3$

Add/Subtract the numbers: $-2+3=1$

$=1$

$=\frac{1}{3}$

$=\frac{(x-2)^{-\frac{2}{3}+1}}{\frac{1}{3}}$

$(x-2)^{-\frac{2}{3}+1}=(x-2)^{\frac{1}{3}}$ *Hide Steps*

$(x-2)^{-\frac{2}{3}+1}$

Join $-\frac{2}{3}+1$: $\frac{1}{3}$ *Hide Steps*

$-\frac{2}{3}+1$

Convert element to fraction: $1=\frac{1\cdot 3}{3}$

$=-\frac{2}{3}+\frac{1\cdot 3}{3}$

Since the denominators are equal, combine the fractions: $\frac{a}{c}\pm\frac{b}{c}=\frac{a\pm b}{c}$

$=\frac{-2+1\cdot 3}{3}$

$-2+1\cdot 3=1$ *Hide Steps*

$-2+1\cdot 3$

Multiply the numbers: $1\cdot 3=3$

$=-2+3$

Add/Subtract the numbers: $-2+3=1$

$=1$

$=\frac{1}{3}$

$=(x-2)^{\frac{1}{3}}$

$=\frac{(x-2)^{\frac{1}{3}}}{\frac{1}{3}}$

Apply the fraction rule: $\frac{a}{\frac{b}{c}}=\frac{a\cdot c}{b}$

$=\frac{(x-2)^{\frac{1}{3}}\cdot 3}{1}$

Apply rule $\frac{a}{1}=a$

$=3(x-2)^{\frac{1}{3}}$

$=3(x-2)^{\frac{1}{3}}$

Add a constant to the solution

$=3(x-2)^{\frac{1}{3}}+C$

Compute the boundaries: $\int_2^4 \frac{1}{(x-2)^{\frac{2}{3}}}dx=3\cdot\ 2^{\frac{1}{3}}-0$ *Hide Steps*

$\int_a^b f(x)dx=F(b)-F(a)=\lim_{x\to\ b-}(F(x))-\lim_{x\to\ a+}(F(x))$

$\lim_{x\to\ 2+}\left(3(x-2)^{\frac{1}{3}}\right)=0$ *Hide Steps*

$\lim_{x\to\ 2+}\left(3(x-2)^{\frac{1}{3}}\right)$

Plug in the value $x=2$

$=3(2-2)^{\frac{1}{3}}$

Simplify

$=0$

Hide Steps

$\lim_{x\to 4-}\left(3(x-2)^{\frac{1}{3}}\right)=3\cdot\ 2^{\frac{1}{3}}$

$\lim_{x\to 4-}\left(3(x-2)^{\frac{1}{3}}\right)$

Plug in the value $x=4$

$=3(4-2)^{\frac{1}{3}}$

Simplify

$=3\cdot\ 2^{\frac{1}{3}}$

$=3\cdot\ 2^{\frac{1}{3}}-0$

$=3\cdot\ 2^{\frac{1}{3}}-0$

Simplify

$=3\cdot\ 2^{\frac{1}{3}}$

$=3+3\cdot\ 2^{\frac{1}{3}}$

Graph

Plotting: $\frac{1}{(x-2)^{\frac{2}{3}}}$