



$$= \frac{x^3}{3}$$

$$= \pi \left( r^2 x - \frac{x^3}{3} \right)$$

Add a constant to the solution

$$= \pi \left( r^2 x - \frac{x^3}{3} \right) + C$$

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Compute the boundaries:  $\int_{-r}^r \pi(r^2 - x^2) dx = \pi \left( r^3 - \frac{r^3}{3} \right) - \pi \left( -r^3 + \frac{r^3}{3} \right)$

$$\int_a^b f(x) dx = F(b) - F(a) = \lim_{x \rightarrow b^-} (F(x)) - \lim_{x \rightarrow a^+} (F(x))$$

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$$\lim_{x \rightarrow -r^+} \left( \pi \left( r^2 x - \frac{x^3}{3} \right) \right) = \pi \left( -r^3 + \frac{r^3}{3} \right)$$

$$\lim_{x \rightarrow -r^+} \left( \pi \left( r^2 x - \frac{x^3}{3} \right) \right)$$

Plug in the value  $x = -r$

$$= \pi \left( r^2(-r) - \frac{(-r)^3}{3} \right)$$

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Simplify  $\pi \left( r^2(-r) - \frac{(-r)^3}{3} \right)$ :  $\pi \left( -r^3 + \frac{r^3}{3} \right)$

$$\pi \left( r^2(-r) - \frac{(-r)^3}{3} \right)$$

Remove parentheses:  $(-a) = -a$

$$= \pi \left( -r^2 r - \frac{(-r)^3}{3} \right)$$

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$$r^2 r = r^3$$

$$r^2 r$$

Apply exponent rule:  $a^b \cdot a^c = a^{b+c}$

$$r^2 r = r^{2+1} = r^3$$

$$= r^3$$

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$$\frac{(-r)^3}{3} = -\frac{r^3}{3}$$

$$\frac{(-r)^3}{3}$$

$$(-r)^3 = -r^3$$

Hide Steps 

$$(-r)^3$$

Apply exponent rule:  $(-a)^n = -a^n$ , if  $n$  is odd

$$(-r)^3 = -r^3$$

$$= -r^3$$

$$= \frac{-r^3}{3}$$

Apply the fraction rule:  $\frac{-a}{b} = -\frac{a}{b}$ 

$$= -\frac{r^3}{3}$$

$$= \pi \left( -r^3 - \left( -\frac{r^3}{3} \right) \right)$$

Apply rule  $-(-a) = a$ 

$$= \pi \left( -r^3 + \frac{r^3}{3} \right)$$

$$= \pi \left( -r^3 + \frac{r^3}{3} \right)$$

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$$\lim_{x \rightarrow r^-} \left( \pi \left( r^2 x - \frac{x^3}{3} \right) \right) = \pi \left( r^3 - \frac{r^3}{3} \right)$$

$$\lim_{x \rightarrow r^-} \left( \pi \left( r^2 x - \frac{x^3}{3} \right) \right)$$

Plug in the value  $x = r$ 

$$= \pi \left( r^2 r - \frac{r^3}{3} \right)$$

Apply exponent rule:  $a^b \cdot a^c = a^{b+c}$ 

$$r^2 r = r^{2+1} = r^3$$

$$= \pi \left( r^3 - \frac{r^3}{3} \right)$$

$$= \pi \left( r^3 - \frac{r^3}{3} \right) - \pi \left( -r^3 + \frac{r^3}{3} \right)$$

$$= \pi \left( r^3 - \frac{r^3}{3} \right) - \pi \left( -r^3 + \frac{r^3}{3} \right)$$

Simplify

$$= \frac{4\pi r^3}{3}$$

Graph

