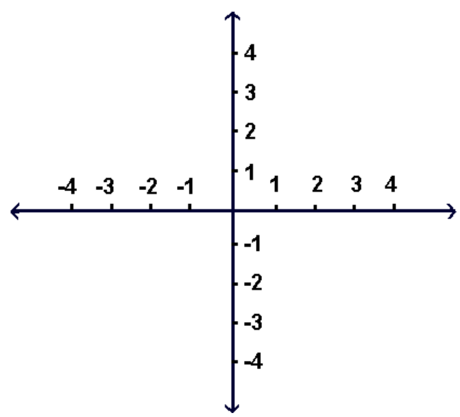
Lecture 3

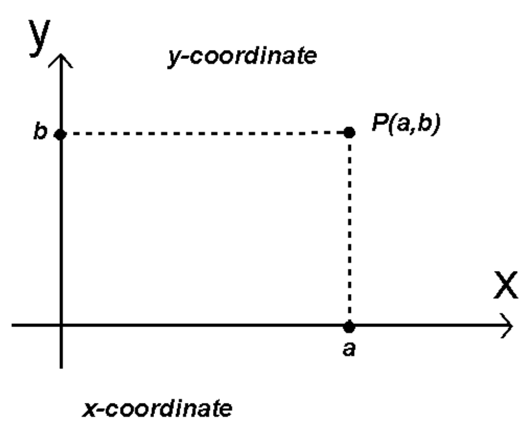
In this lecture we will discuss

- **Graphs in the coordinate plane.**
- **Intercepts.**
- **Symmetry Plane.**

We begin with the Coordinate plane. Just as points on a line can be placed in one-to-one correspondence with the real numbers, so points in the PLANE can be placed in one-to-one correspondence with pairs of real numbers. What is a plane?

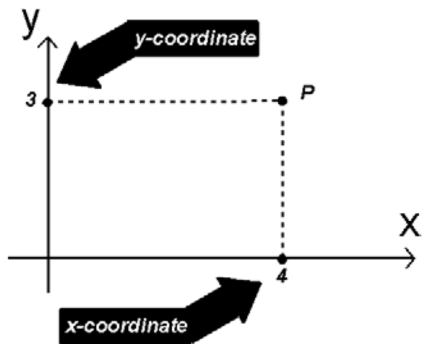


A PLANE is just the intersection of two COORDINATE lines at 90 degrees. It is technically called the COORDINATE PLANE, but we will call it the plane also whenever it is convenient. Each line is a line with numbers on it, so to define a point in the PLANE, we just read of the corresponding points on each line. For example I pick a point in the plane

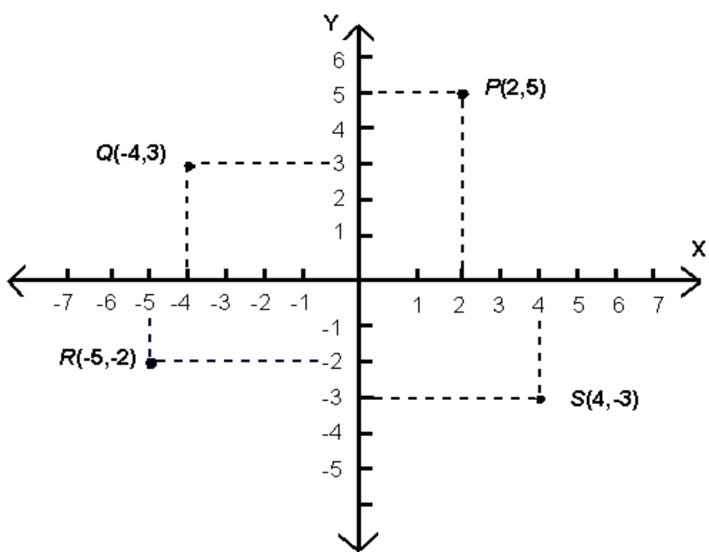


By an ordered pair of real numbers we mean two real numbers in an assigned order. Every point P in a coordinate plane can be associated with a unique ordered pair of real numbers by drawing two lines through P, one perpendicular to the x-axis and the other to the y-axis.

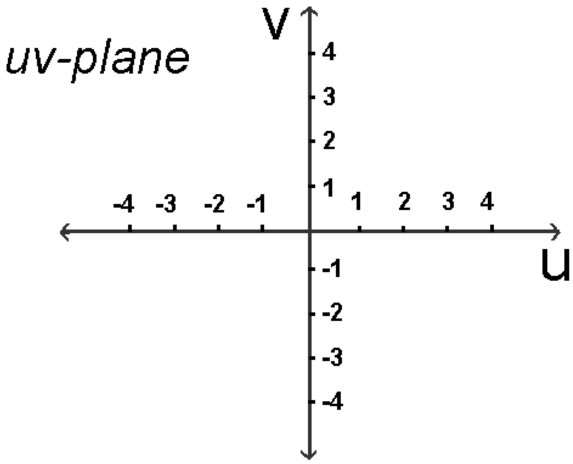
For example if we take (a,b)=(4,3), then on coordinate plane

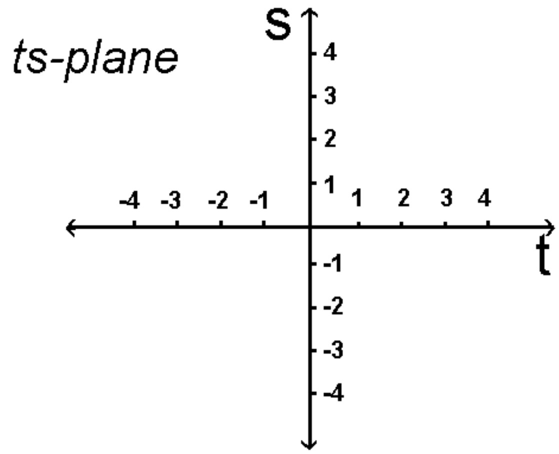


To plot a point *P(a, b)* means to locate the point with coordinates *(a, b)* in a coordinate plane. For example, In the figure below we have plotted the points P(2,5), Q(-4,3), R(-5,-2), and S(4,-3).Now this idea will enable us to visualise algebraic equations as geometric curves and, conversely, to represent geometric curves by algebraic equations.

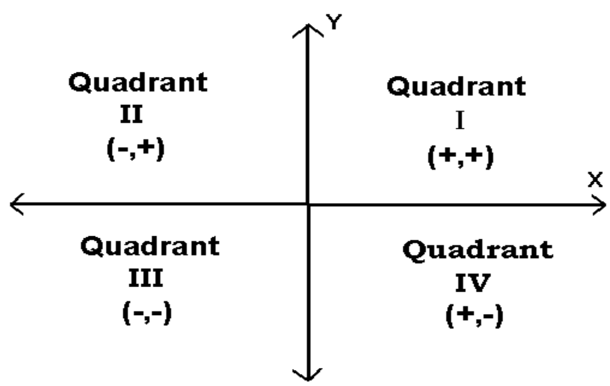


Labelling the axes with letters *x* and *y* is a common convention, but any letters may be used. If the letters x and y are used to label the coordinate axes, then the resulting plane is also called an *xy-plane*. In applications it is common to use letters other than *x* and *y* to label coordinate axes. Figure below shows a uv-plane and a ts-plane. The first letter in the name of the plane refers to the horizontal axis and the second to the vertical axis.





Here is another terminology. The COORDINATE PLANE and the ordered pairs we just discussed is together known as the RECTANGULAR COORDINATE SYSTEM. In a rectangular coordinate system the coordinate axes divide the plane into four regions called *quadrants*. These are numbered counter clockwise with Roman numerals as shown in the Figure below.



Consider the equations

$$5xy = 2$$
$$x^2 + 2y^2 = 7$$
$$y = x^3 - 7$$

We define a solution of such an equation to be an ordered pair of real numbers(a,b) so that the equation is satisfactory when we substitute x=a and y=b.

Example 1

The pair (3,2) is a solution of

$$6x - 4y = 10$$

since this equation is satisfied when we substitute x = 3 and y = 2. That is

$$6(3)-4(2)=10$$

which is true!!

However, the pair (2,0) is not a solution, since

$$6(2)-4(0)=18\neq 10$$

We make the following definition in order to start seeing algebraic objects geometrically.

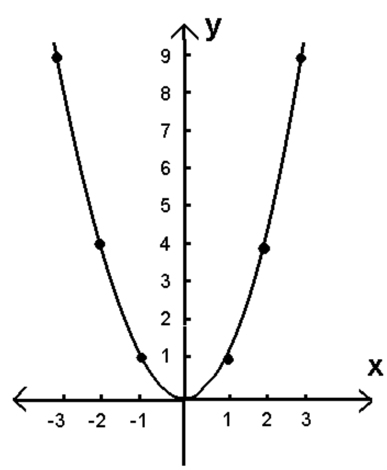
Definition. The GRAPH of an equation in two variables x and y is the set of all points in the xy-plane whose coordinates are members of the solution set of the equation.

Example 2

Sketch the graph of $y = x^2$

x	$y = x^2$	(x , y)
0	0	(0 , 0)
1	1	(1 , 1)
2	4	(2 , 4)
3	9	(3 , 9)
-1	1	(-1 , 1)
-2	4	(-2 , 4)
-3	9	(-3 , 9)

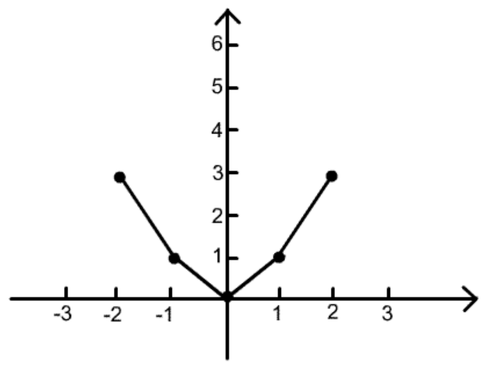
When we plot these on the xy-plane and connect them, we get this picture of the graph



IMPORTANT REMARK.

It should be kept in mind that the curve in above is only an approximation to the graph of $y = x^2$.

When a graph is obtained by plotting points, whether by hand, calculator, or computer, there is no guarantee that the resulting curve has the correct shape. For example, the curve in the Figure here pass through the points tabulated in above table.

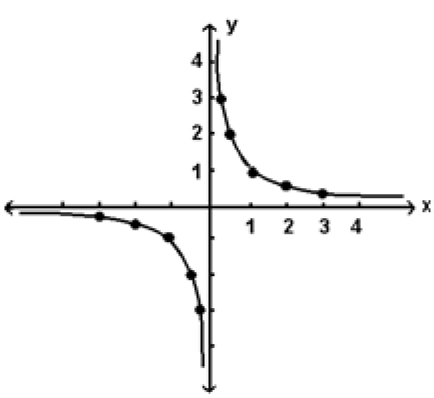


Example : Sketch the graph of

y = 1/x

x	y = 1/x	(x , y)
1/3	3	(1/3 , 3)
1/2	2	(1/2 , 2)
1	1	(1 , 1)
2	1/2	(2 , 1/2)
3	1/3	(3 , 1/3)
-1/3	-3	(-1/3 , -3)
-1/2	-2	(-1/2 , -2)
-1	-1	(-1 , -1)
-2	-1/2	(-2, -1/2)
-3	-1/3	(-3 , -1/3)

Because 1/x is undefined when x=0, we can plot only points for which x=0



INTERCEPTS

Points where a graph intersects the coordinate axes are of special interest in many problems. As illustrated before, intersections of a graph with the x-axis have the form (a, 0) and intersections with the y-axis have the form (0, b). The number a is called an x-intercept of the graph and the number b a y-intercept.

Example: Find all intercepts of

(a) $3x + 2y = 6$

(b) $x = y^2 - 2y$

(c) $y = 1/x$

Solution

$3x + 2y = 6$

x-intercepts

Set $y = 0$ and solve for x

$3x = 6$ or **$x = 2$**

is the required x-intercept

$3x + 2y = 6$

y-intercepts

Set $x = 0$ and solve for y

$2y = 6$ or **$y = 3$**

is the required y-intercept

Similarly you can solve part (b), the part (c) is solved here

$y = 1/x$

x-intercepts

Set $y = 0$

$1/x = 0 \Rightarrow$ **x** is undefined

No x-intercept

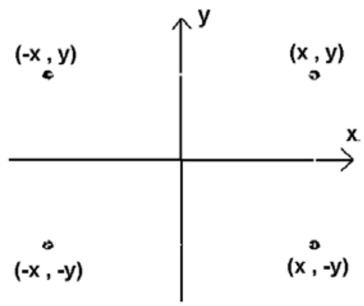
y-intercepts

Set $x = 0$

$y = 1/0 \Rightarrow$ **y** is undefined

No y-intercept

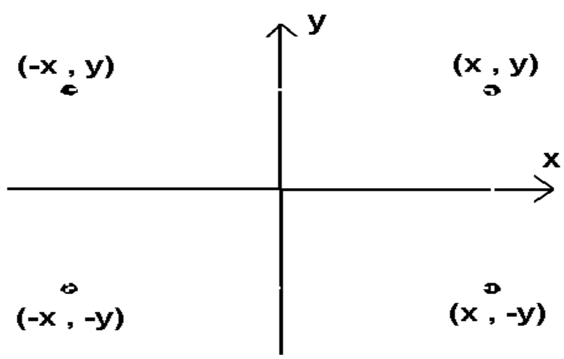
In the following figure, the points (x,y),(-x,y),(x,-y) and (-x,-y) form the corners of a rectangle.

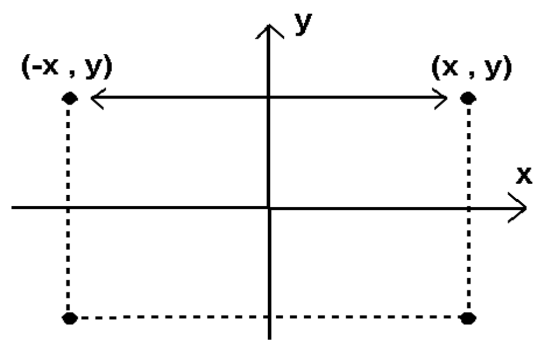


SYMMETRY

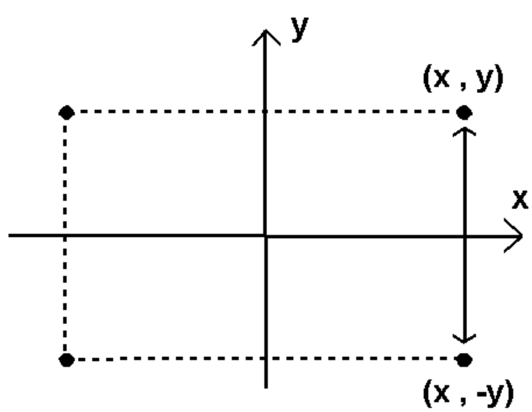
Symmetry is at the heart of many mathematical arguments concerning the structure of the universe, and certainly symmetry plays an important role in applied mathematics and engineering fields. Here is what it is.

As illustrated in Figure the points





Symmetric about the y-axis



Symmetric about the x-axis

$(x, y),\ (-x, y),\ (x, -y)$ *and* $(-x, -y)$ form the corners of a rectangle.

For obvious reasons, the points (x,y) and (x,-y) are said to be symmetric about the x-axis and the points

(x, y) and (-x, y) are symmetric about the y-axis and the points (x, y) and (-x, -y) symmetric about the origin.

SYMMETRY AS A TOOL FOR GRAPHING

By taking advantage of symmetries when they exist, the work required to obtain a graph can be reduced considerably.

Example 9

Sketch the graph of the equation

$$y=\frac{1}{8}x^4-x^2$$

Solution. The graph is symmetric about the y-axis since substituting $-x$ for x yields $y=\frac{1}{8}(-x)^4-(-x)^2$

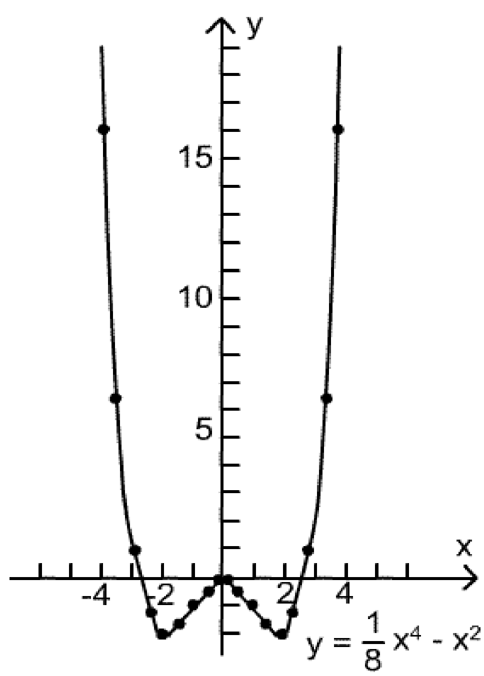
which simplifies to the original equation.

As a consequence of this symmetry, we need only calculate points on the graph that lies in the right half of the xy-plane (x >= 0).

The corresponding points in the left half of the xy-plane (x <= 0).

can be obtained with no additional computation by using the symmetry. So put only positive x-values in given equation and evaluate corresponding y-values.

Since graph is symmetric about y-axis, we will just put negative signs with the x-values taken before and take the same y-values as evaluated before for positive x-values.

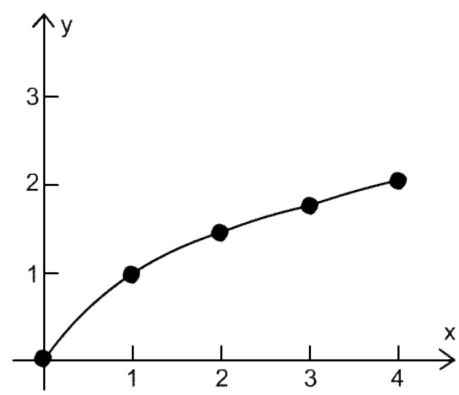


Example 10 Sketch the graph of $x = y^2$

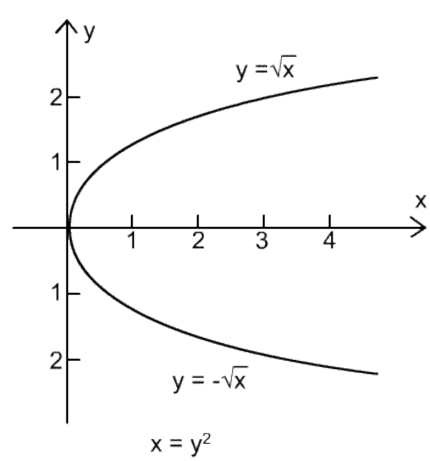
Solution. If we solve $x = y^2$ for y in terms of x, we obtain two solutions,

$y = \sqrt{x}$ and $y = -\sqrt{x}$

The graph of $y = \sqrt{x}$ is the portion of the curve $x = y^2$ that lies above or touches the x-axis (since $y = \sqrt{x} \geq 0$), and the graph of $y = -\sqrt{x}$ is the portion that lies below or touches the x-axis (since $y = -\sqrt{x} \leq 0$). However, the curve $x = y^2$ is symmetric about the x-axis because substituting $-y$ for *y* yields $x = (-y)^2$ which is equivalent to the original equation. Thus, we need only graph $y = \sqrt{x}$ and then reflect it about the x-axis to complete the graph $x = y^2$.



is the graph of the function. $y = \sqrt{x}$



is the required graph of the function.