MTH101 Solution: Practice Questions
Lecture No.16: Techniques of Differentiation

Q.No.1: Differentiate $g(t)=\dfrac{t^2+4}{2t}$.

Solution:

$\because\ g(t)=\dfrac{t^2+4}{2t},$

$$\therefore g'(t)=\frac{2t\frac{d}{dt}(t^2+4)-(t^2+4)\frac{d}{dt}(2t)}{(2t)^2},\quad (\because \text{ quotient rule})$$

$$=\frac{2t(2t)-(t^2+4)(2)}{4t^2}$$

$$=\frac{4t^2-2t^2-8}{4t^2}$$

$$=\frac{2t^2-8}{4t^2}$$

$$=\frac{t^2-4}{2t^2}.$$

Q.No.2: Evaluate $\dfrac{d}{dx}\left((x+1)(1+\sqrt{x})\right)$ at $x=9$.

Solution:

$$\frac{d}{dx}\left((x+1)(1+\sqrt{x})\right)=(x+1)\frac{d}{dx}(1+\sqrt{x})+(1+\sqrt{x})\frac{d}{dx}(x+1),\quad (\because \text{ product rule})$$

$$=(x+1)\left(\frac{1}{2\sqrt{x}}\right)+(1+\sqrt{x})(1),$$

$$=\frac{(x+1)}{2\sqrt{x}}+(1+\sqrt{x}),$$

by substituting $x=9$, $=\dfrac{(9+1)}{2\sqrt{9}}+(1+\sqrt{9})=\dfrac{10}{6}+4=\dfrac{10+24}{6}=\dfrac{34}{6}=\dfrac{17}{3}.$

Q.No.3: Differentiate the following functions:

- **i.** $h(x)=(2x+1)(x+\sqrt{x})$.
- **ii.** $g(x)=x^{-3}(5x^{-4}+3)$.
- **iii.** $f(x)=\dfrac{x^3+1}{4x^2+1}$.

Solution (i): $h(x)=(2x+1)(x+\sqrt{x})$.

$$\frac{d}{dx}(h(x)) = (2x+1)\frac{d}{dx}(x+\sqrt{x}) + (x+\sqrt{x})\frac{d}{dx}(2x+1), \quad (\because \text{ product rule})$$

$$= (2x+1)(1+\frac{1}{2\sqrt{x}}) + (x+\sqrt{x})(2),$$

$$= (2x+1)\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right) + (2x+2\sqrt{x}),$$

$$= 2x+1+\sqrt{x}+\frac{1}{2\sqrt{x}}+2x+2\sqrt{x},$$

$$= 4x+3\sqrt{x}+\frac{1}{2\sqrt{x}}+1.$$

Solution (ii): $\text{g}(x) = x^{-3}(5x^{-4}+3).$

$\because \quad g(x) = x^{-3}(5x^{-4}+3) = 5x^{-7}+3x^{-3},$

$$\therefore \quad \frac{d}{dx}(g(x)) = 5\frac{d}{dx}(x^{-7}) + 3\frac{d}{dx}(x^{-3}),$$

$$= 5(-7x^{-8}) + 3(-3x^{-4}),$$

$$= -35x^{-8} - 9x^{-4}.$$

Solution (iii): $f(x) = \dfrac{x^3+1}{4x^2+1}.$

$\because \quad f(x) = \dfrac{x^3+1}{4x^2+1},$

$$\therefore \frac{d}{dx}(f(x)) = \frac{(4x^2+1)\frac{d}{dx}(x^3+1) - (x^3+1)\frac{d}{dx}(4x^2+1)}{(4x^2+1)^2}, \quad (\because \text{ quotient rule})$$

$$= \frac{(4x^2+1)(3x^2) - (x^3+1)(8x)}{(4x^2+1)^2},$$

$$= \frac{12x^4+3x^2-(8x^4+8x)}{(4x^2+1)^2},$$

$$= \frac{4x^4+3x^2-8x}{(4\,\text{x}^2+1)^2}.$$

MTH101 Solution: Practice Questions
Lecture No.17: Derivatives of Trigonometric Function

Q.No.1: Find $\frac{dy}{dx}$ if $y = x^3 \cot x - \frac{3}{x^3}$.

Answer: $3x^2 \cot x - x^3 cosec^2 x + \frac{9}{x^4}$

Solution:

Given $y = x^3 \cot x - \frac{3}{x^3}$,

$\frac{dy}{dx} = \cot x \frac{d}{dx}(x^3) + x^3 \frac{d}{dx}(\cot x) - \frac{d}{dx}\left(\frac{3}{x^3}\right)$,

$= \cot x\,(3x^2) + x^3(-cosec^2 x) - 3\frac{d}{dx}\left(\frac{1}{x^3}\right)$,

$= 3x^2 \cot x - x^3 cosec^2 x + \frac{9}{x^4}$ (Answer).

Q.No.2: Find $\frac{dy}{dx}$ if $y = x^4 \sin x$ at $x = \pi$.

Answer: $-\pi^4$

Solution:

$\because \frac{d}{dx}(f.g) = f.\frac{d}{dx}(g) + g.\frac{d}{dx}(f)$,

$y = x^4 \sin x$ at $x = \pi$,

$\frac{d}{dx} = \sin x \frac{d}{dx}(x^4) + x^4 \frac{d}{dx}(\sin x)$,

$= \sin x\ (4x^3) + x^4(\cos x)$,

$= 4x^3 \sin x + x^4 \cos x$,

$= 4\pi^3 \sin \pi + \pi^4 \cos \pi$, at $x = \pi$,

$= 4\pi^3(0) + \pi^4(-1)$,

$= -\pi^4$ (Answer).

Q.No.3: Find $f'(t)$ if $f(t) = \frac{2-8t+t^2}{sint}$.

Answer: $\frac{[(2t-8)(\sin t)]-[(t^2-8t+2)(\cos t)]}{\sin^2 t}$

Solution:

$\because \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).\frac{d}{dx}(f(x)) - f(x).\frac{d}{dx}(g(x))}{[g(x)]^2}$,

$f(t) = \frac{2-8t+t^2}{sint}$,

$f'(t) = \frac{[(\sin t)(-8+2t)]-[(2-8t+t^2)(\cos t)]}{(\sin t)^2}$,

$= \frac{[(2t-8)(\sin t)]-[(t^2-8t+2)(\cos t)]}{\sin^2 t}$ (Answer).

Q.No.4: Find $f'(y)$ if $(y) = \frac{\sin y + 3\tan y}{y^3 - 2}$.

Answer: $\frac{[(y^3-2)(\cos y+3\sec^2 y)]-[(\sin y+3\tan y)+(3y^2)]}{y^6-4y^3+4}$

Solution:

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).\frac{d}{dx}(f(x)) - f(x).\frac{d}{dx}(g(x))}{[g(x)]^2}$,
$f(y) = \frac{\sin y + 3\tan y}{y^3 - 2}$.
$f'(y) = \frac{[(y^3-2)(\cos y + 3\sec^2 y)] - [(\sin y + 3\tan y) + (3y^2)]}{(y^3-2)^2}$,
$= \frac{[(y^3-2)(\cos y + 3\sec^2 y)] - [(\sin y + 3\tan y) + (3y^2)]}{y^6 - 4y^3 + 4}$ (Answer).

Q.No.5: (a) Find $\frac{dy}{dx}$ if $y = (5x^2 + 3x + 3)(\sin x)$.
(b) Find $f'(t)$ if $(t) = 5t\sin t$.
Answer: (a) $(5x^2 + 3x + 3)(\cos x) + \sin x\,.(10x + 3)$
(b) $5t\cos t + 5\sin t$

Solution:

(a) $\because \frac{d}{dx}(f.g) = f.\frac{d}{dx}(g) + g.\frac{d}{dx}(f)$,
$y = (5x^2 + 3x + 3)(\sin x)$,
$\frac{d}{dx}[(5x^2 + 3x + 3)(\sin x)] = (5x^2 + 3x + 3)(\cos x) + \sin x\,(10x + 3)$ (Answer).

(b) $\because \frac{d}{dx}(f.g) = f.\frac{d}{dx}(g) + g.\frac{d}{dx}(f)$,
$f(t) = 5t\sin t$,
$\frac{d}{dx}(5t\sin t) = 5t\cos t + (\sin t)(5)$,
$= 5t\cos t + 5\sin t$ (Answer).

MTH101 Solution: Practice Questions
Lecture No.18: The Chain Rule

Q.No.1: Differentiate $y=\sqrt{5x^3-3x^2+x}$ with respect to "x" using the chain rule.

Solution:

Given function is $y=\sqrt{5x^3-3x^2+x}$.

Let $u=5x^3-3x^2+x.$

Then $y=\sqrt{u}$.

Using chain rule, $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}.$

Here,

$$\frac{dy}{du}=\frac{1}{2}u^{-\frac{1}{2}}=\frac{1}{2\sqrt{u}},$$

$$\frac{du}{dx}=15x^2-6x+1.$$

Then,

$$\frac{dy}{dx}=\frac{1}{2\sqrt{u}}(15x^2-6x+1),$$

$$\therefore \frac{dy}{dx}=\frac{1}{2\sqrt{5x^3-3x^2+x}}(15x^2-6x+1.$$

Q.No.2: Differentiate $\mathrm{y}=\tan\sqrt{x}+\cos\sqrt{x}$ with respect to "x" using the chain rule.

Solution:

Given function is $y=\tan\sqrt{x}+\cos\sqrt{x}.$

Let $u=\sqrt{x}.$

Then $y=\tan(u)+\cos(u).$

Using chain rule, $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$

Here,

$$\frac{dy}{du}=\sec^2 u-\sin u,$$

$$\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}.$$

Then,

$$\frac{dy}{dx} = (\sec^2 u - \sin u).\frac{1}{2\sqrt{x}},$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x}}\left(\sec^2 \sqrt{x} - \sin \sqrt{x}\right).$$

Q.No.3: Differentiate $y = 3\sin^2 x^5 + 4\cos^2 x^5$ with respect to "x" using the chain rule.

Solution:

Given function is $y = 3\sin^2 x^5 + 4\cos^2 x^5$.

Let $u = x^5$.

Then $y = 3\sin^2 u + 4\cos^2 u$.

Using chain rule, $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$.

Here,

$$\frac{dy}{du} = 3\times 2\sin u\cos u + 4\times 2\cos u(-\sin u),$$

$$= 6\sin u\cos u - 8\cos u\sin u,$$

$$= -2\sin u\cos u,$$

$$\frac{du}{dx} = 5x^4.$$

Then,

$$\frac{dy}{dx} = 5x^4(-2\cos u\sin u),$$

$$\therefore \frac{dy}{dx} = -10x^4(\cos x^5 \sin x^5).$$

Q.No.4: Find $\frac{dy}{dx}$ if $y = \sqrt{\sec 4x}$ using chain rule.

Solution:

Given function is $y = \sqrt{\sec 4x}$.

Let $u = \sec 4x$.

Then $y = \sqrt{u}$.

Using chain rule, $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$.

Here,

$$\frac{dy}{du}=\frac{1}{2}u^{-\frac{1}{2}}=\frac{1}{2\sqrt{u}},$$

$$\frac{du}{dx}=4\sec 4x\tan 4x.$$

Then,

$$\frac{dy}{dx}=\frac{1}{2\sqrt{u}}(4\sec 4x\tan 4x),$$

$$\therefore \frac{dy}{dx}=\frac{1}{2\sqrt{\sec 4x}}(4\sec 4x\tan 4x),$$

$$=2\sqrt{\sec 4x}\ \tan 4x.$$

Q.No.5: Find $\frac{dy}{dt}$ if $y=\tan t^{\frac{2}{3}}$ using chain rule.

Solution:

Given function is $y=\tan t^{\frac{2}{3}}$.

Let $u=t^{\frac{2}{3}}$.

Then $y=\tan u$.

Using chain rule, $\frac{dy}{dt}=\frac{dy}{du}.\frac{du}{dt}$.

Here,

$$\frac{dy}{du}=\sec^2 u,$$

$$\frac{du}{dt}=\frac{2}{3}t^{-\frac{1}{3}}.$$

Then,

$$\frac{dy}{dt}=\sec^2 u\left(\frac{2}{3}t^{-\frac{1}{3}}\right),$$

$$\therefore \frac{dy}{dt}=\frac{2}{3t^{\frac{1}{3}}}\sec^2 t^{\frac{2}{3}}.$$