Solution

$\int_0^2 \pi\left(\left(\frac{1}{2}+x^2\right)^2-x^2\right)dx=\frac{69\pi}{10}$ (Decimal: $21.67699\ldots$)

Steps

$\int_0^2 \pi\left(\left(\frac{1}{2}+x^2\right)^2-x^2\right)dx$

Hide Steps

Compute the indefinite integral: $\int \pi\left(\left(\frac{1}{2}+x^2\right)^2-x^2\right)dx=\pi\left(\frac{x^5}{5}+\frac{1}{4}x\right)+C$

$\int \pi\left(\left(\frac{1}{2}+x^2\right)^2-x^2\right)dx$

Take the constant out: $\int a\cdot f(x)dx=a\cdot\int f(x)dx$

$=\pi\cdot\int\left(\frac{1}{2}+x^2\right)^2-x^2dx$

Hide Steps

Expand $\left(\frac{1}{2}+x^2\right)^2-x^2$: $x^4+\frac{1}{4}$

$\left(\frac{1}{2}+x^2\right)^2-x^2$

Hide Steps

$\left(\frac{1}{2}+x^2\right)^2$: $\frac{1}{4}+x^2+x^4$

Apply Perfect Square Formula: $(a+b)^2=a^2+2ab+b^2$

$a=\frac{1}{2},\ b=x^2$

$=\left(\frac{1}{2}\right)^2+2\cdot\frac{1}{2}x^2+\left(x^2\right)^2$

Hide Steps

Simplify $\left(\frac{1}{2}\right)^2+2\cdot\frac{1}{2}x^2+\left(x^2\right)^2$: $\frac{1}{4}+x^2+x^4$

$\left(\frac{1}{2}\right)^2+2\cdot\frac{1}{2}x^2+\left(x^2\right)^2$

Hide Steps

$\left(\frac{1}{2}\right)^2=\frac{1}{4}$

$\left(\frac{1}{2}\right)^2$

Apply exponent rule: $\left(\frac{a}{b}\right)^c=\frac{a^c}{b^c}$

$=\frac{1^2}{2^2}$

Apply rule $1^a=1$

$1^2=1$

$=\frac{1}{2^2}$

$2^2=4$

$=\frac{1}{4}$

$2\cdot\frac{1}{2}x^2=x^2$

Hide Steps

$2\cdot\frac{1}{2}x^2$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot 2x^2}{2}$

Cancel the common factor: 2

$=1\cdot x^2$

Multiply: $1\cdot x^2=x^2$

$=x^2$

$(x^2)^2=x^4$

Hide Steps

$(x^2)^2$

Apply exponent rule: $(a^b)^c=a^{bc}$

$=x^{2\cdot 2}$

Refine

$=x^4$

$=\frac{1}{4}+x^2+x^4$

$=\frac{1}{4}+x^2+x^4$

$=\frac{1}{4}+x^2+x^4-x^2$

Simplify $\frac{1}{4}+x^2+x^4-x^2$: $x^4+\frac{1}{4}$

Hide Steps

$\frac{1}{4}+x^2+x^4-x^2$

Group like terms

$=x^4+x^2-x^2+\frac{1}{4}$

Add similar elements: $x^2-x^2=0$

$=x^4+\frac{1}{4}$

$=x^4+\frac{1}{4}$

$=\pi\cdot\int x^4+\frac{1}{4}dx$

Apply the Sum Rule: $\int f(x)\pm g(x)dx=\int f(x)dx\pm\int g(x)dx$

$=\pi\left(\int x^4dx+\int\frac{1}{4}dx\right)$

$\int x^4dx=\frac{x^5}{5}$ Hide Steps

$\int x^4dx$

Apply the Power Rule: $\int x^adx=\frac{x^{a+1}}{a+1},\quad a\neq-1$

$=\frac{x^{4+1}}{4+1}$

Simplify

$=\frac{x^5}{5}$

$\int\frac{1}{4}dx=\frac{1}{4}x$ Hide Steps

$\int\frac{1}{4}dx$

Integral of a constant: $\int adx=ax$

$=\frac{1}{4}x$

$=\pi\left(\frac{x^5}{5}+\frac{1}{4}x\right)$

Add a constant to the solution

$=\pi\left(\frac{x^5}{5}+\frac{1}{4}x\right)+C$

Hide Steps

Compute the boundaries: $\int_0^2 \pi\left(\left(\frac{1}{2}+x^2\right)^2-x^2\right)dx=\pi\frac{69}{10}-0$

$\int_a^b f(x)dx=F(b)-F(a)=\lim_{x\to b-}(F(x))-\lim_{x\to a+}(F(x))$

Hide Steps

$\lim_{x\to 0+}\left(\pi\left(\frac{x^5}{5}+\frac{1}{4}x\right)\right)=0$

$\lim_{x\to 0+}\left(\pi\left(\frac{x^5}{5}+\frac{1}{4}x\right)\right)$

Plug in the value $x=0$

$=\pi\left(\frac{0^5}{5}+\frac{1}{4}\cdot 0\right)$

Hide Steps

Simplify $\pi\left(\frac{0^5}{5}+\frac{1}{4}\cdot 0\right)$: 0

$\pi\left(\frac{0^5}{5}+\frac{1}{4}\cdot 0\right)$

Apply rule $0^a=0$

$0^5=0$

$=\pi\left(0\cdot\frac{1}{4}+\frac{0}{5}\right)$

Hide Steps

Simplify $\frac{0}{5}+0\cdot\frac{1}{4}$: $0\cdot\frac{1}{4}+0$

$\frac{0}{5}+0\cdot\frac{1}{4}$

Apply rule $\frac{0}{a}=0, a\neq 0$

$=0+\frac{1}{4}\cdot 0$

$=\pi\left(0\cdot\frac{1}{4}+0\right)$

Apply rule $0\cdot a=0$

$=\pi(0+0)$

Add the numbers: $0+0=0$

$=0\pi$

Apply rule $0\cdot a=0$

$=0$

$=0$

Hide Steps

$\lim_{x\to 2-}\left(\pi\left(\frac{x^5}{5}+\frac{1}{4}x\right)\right)=\pi\frac{69}{10}$

$\lim_{x\to 2-}\left(\pi\left(\frac{x^5}{5}+\frac{1}{4}x\right)\right)$

Plug in the value $x=2$

$=\pi\left(\frac{2^5}{5}+\frac{1}{4}\cdot\ 2\right)$

Simplify $\pi\left(\frac{2^5}{5}+\frac{1}{4}\cdot\ 2\right)$: $\pi\frac{69}{10}$ *Hide Steps*

$\pi\left(\frac{2^5}{5}+\frac{1}{4}\cdot\ 2\right)$

$\frac{1}{4}\cdot\ 2=\frac{1}{2}$ *Hide Steps*

$\frac{1}{4}\cdot\ 2$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot\ b}{c}$

$=\frac{1\cdot\ 2}{4}$

Multiply the numbers: $1\cdot\ 2=2$

$=\frac{2}{4}$

Cancel the common factor: 2

$=\frac{1}{2}$

$=\pi\left(\frac{1}{2}+\frac{2^5}{5}\right)$

Join $\frac{2^5}{5}+\frac{1}{2}$: $\frac{69}{10}$ *Hide Steps*

$\frac{2^5}{5}+\frac{1}{2}$

Least Common Multiplier of $5, 2$: 10 *Hide Steps*

$5, 2$

Least Common Multiplier (LCM)

The LCM of a, b is the smallest positive number that is divisible by both a and b

Prime factorization of 5: 5 *Hide Steps*

5

5 is a prime number, therefore no factorization is possible

$=5$

Prime factorization of 2: 2

Hide Steps

2

2 is a prime number, therefore no factorization is possible

$=2$

Multiply each factor the greatest number of times it occurs in either 5 or 2

$=5\cdot 2$

Multiply the numbers: $5\cdot 2=10$

$=10$

Adjust Fractions based on the LCM

Hide Steps

Multiply each numerator by the same amount needed to multiply its corresponding denominator to turn it into the LCM 10

For $\frac{2^5}{5}$: multiply the denominator and numerator by 2

$\frac{2^5}{5}=\frac{2^5\cdot 2}{5\cdot 2}=\frac{64}{10}$

For $\frac{1}{2}$: multiply the denominator and numerator by 5

$\frac{1}{2}=\frac{1\cdot 5}{2\cdot 5}=\frac{5}{10}$

$=\frac{64}{10}+\frac{5}{10}$

Since the denominators are equal, combine the fractions: $\frac{a}{c}\pm\frac{b}{c}=\frac{a\pm b}{c}$

$=\frac{64+5}{10}$

Add the numbers: $64+5=69$

$=\frac{69}{10}$

$=\pi\frac{69}{10}$

$=\pi\frac{69}{10}$

$=\pi\frac{69}{10}-0$

$=\pi\frac{69}{10}-0$

Simplify

$$= \frac{69\pi}{10}$$

Graph

Plotting: $\pi\left(\left(\frac{1}{2}+x^2\right)^2-x^2\right)$