<u>LECTURE # 6</u>

Sets

A well defined collection of {distinct}objects is called a set.

- The objects are called the elements or members of the set.
- Sets are denoted by capital letters A, B, C …, X, Y, Z.
- The elements of a set are represented by lower case letters a, b, c, … , x, y, z.
- If an object x is a member of a set A we write x□A, which reads "x belongs to A" or "x is in A" or "x is an element of A", otherwise we write x□A, which reads "x does not belong to A" or "x is not in A" or "x is not an element of A".

TABULAR FORM

Listing all the elements of a set, separated by commas and enclosed within braces or curly brackets{}.

EXAMPLES

In the following examples we write the sets in Tabular Form.
A = {1, 2, 3, 4, 5} is the set of first five **Natural Numbers**.
B = {2, 4, 6, 8, …, 50} is the set of **Even numbers** up to 50.
C = {1, 3, 5, 7, 9 …} is the set of **positive odd numbers.**

NOTE

The symbol "…" is called an ellipsis. It is a short for "and so forth."

<u>*DESCRIPTIVE FORM:*</u>

Stating in words the elements of a set.

<u>EXAMPLES</u>

Now we will write the same examples which we write in Tabular Form ,in the Descriptive Form.

A = set of first five Natural Numbers.(is the Descriptive Form)
B = set of positive even integers less or equal to fifty.
(is the Descriptive Form)
C = {1, 3, 5, 7, 9, …} (is the Descriptive Form)
C = set of positive odd integers. (is the Descriptive Form)

SET BUILDER FORM:

Writing in symbolic form the common characteristics shared by all the elements of the set.

<u>EXAMPLES:</u>

Now we will write the same examples which we write in Tabular as well as Descriptive Form ,in Set Builder Form .

A = {x ÎN / x<=5} (is the Set Builder Form)
B = {x Î E / 0 < x <=50} (is the Set Builder Form)
C = {x ÎO / 0 < x } (is the Set Builder Form)

<u>SETS OF NUMBERS:</u>

1. **Set of Natural Numbers**

   N = {1, 2, 3, … }

2. **Set of Whole Numbers**

W = {0, 1, 2, 3, … }

3. **Set of Integers**

Z = {…, -3, -2, -1, 0, +1, +2, +3, …}

= {0, ±1, ±2, ±3, …}

{"Z" stands for the first letter of the German word for integer: Zahlen.}

4. **Set of Even Integers**

E = {0, ± 2, ± 4, ± 6, …}

5. **Set of Odd Integers**

O = {± 1, ± 3, ± 5, …}

6. **Set of Prime Numbers**

P = {2, 3, 5, 7, 11, 13, 17, 19, …}

7. **Set of Rational Numbers (or Quotient of Integers)**

Q = {x | x = ; p, q □Z, q ≠ 0}

8. **Set of Irrational Numbers**

Q = Q□ = { x | x is not rational}

For example, √2, √3, π, e, etc.

9. **Set of Real Numbers**

R = Q ∪ Q□

10. **Set of Complex Numbers**

C = {z | z = x + *i*y; x, y □ R}

SUBSET:

If A & B are two sets, A is called a subset of B, written A ⊆ B, if, and only if, any element of A is also an element of B.

Symbolically:

A ⊆ B ⇔ if x □ A then x □ B

REMARK:

1. When A ⊆ B, then B is called a superset of A.
2. When A is not subset of B, then there exist at least one x □ A such that x ∉B.
3. Every set is a subset of itself.

EXAMPLES:

Let

A = {1, 3, 5} B = {1, 2, 3, 4, 5}

C = {1, 2, 3, 4} D = {3, 1, 5}

Then

A ⊆ B (Because every element of A is in B)

C ⊆ B (Because every element of C is also an element of B)

A ⊆ D (Because every element of A is also an element of D and also note that every element of D is in A so D ⊆ A)

and A is not subset of C .

(Because there is an element 5 of A which is not in C)

EXAMPLE:

The set of integers "Z" is a subset of the set of Rational Number "Q", since every integer 'n' could be written as:

$$n = \frac{n}{1} \in Q$$

Hence Z ⊆ Q.

PROPER SUBSET

Let A and B be sets. A is a proper subset of B, if, and only if, every element of A is in B but there is at least one element of B that is not in A, and is denoted as $A \subset B$.

EXAMPLE:

Let A = {1, 3, 5} B = {1, 2, 3, 5}

then $A \subset B$ (Because there is an element 2 of B which is not in A).

EQUAL SETS:

Two sets A and B are equal if, and only if, every element of A is in B and every element of B is in A and is denoted A = B.

Symbolically:

$$A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A$$

EXAMPLE:

Let A = {1, 2, 3, 6} B = the set of positive divisors of 6

C = {3, 1, 6, 2} D = {1, 2, 2, 3, 6, 6, 6}

Then A, B, C, and D are all equal sets.

NULL SET:

A set which contains no element is called a **null set**, or an **empty set** or a **void set**. It is denoted by the Greek letter $\varnothing$ (phi) or { }.

EXAMPLE

A = {x | x is a person taller than 10 feet} = $\varnothing$ (Because there does not exist any human being which is taller then 10 feet)

B = {x | x2 = 4, x is odd} = $\varnothing$ (Because we know that there does not exist any odd whose square is 4)

REMARK

$\varnothing$ is regarded as a subset of every set.

EXERCISE:

Determine whether each of the following statements is true or false.

a. $x \in \{x\}$ **TRUE**

(Because x is the member of the singleton set { x })

a. $\{x\} \subseteq \{x\}$ **TRUE**

(Because Every set is the subset of itself.

Note that every Set has necessarily tow subsets $\varnothing$ and the Set itself, these two subset are known as Improper subsets and any other subset is called Proper Subset)

a. $\{x\} \in \{x\}$ **FALSE**

(Because { x} is not the member of {x}) Similarly other

d. $\{x\} \in \{\{x\}\}$ **TRUE**

e. $\varnothing \subseteq \{x\}$ **TRUE**

f. $\varnothing \in \{x\}$ **FALSE**

UNIVERSAL SET:

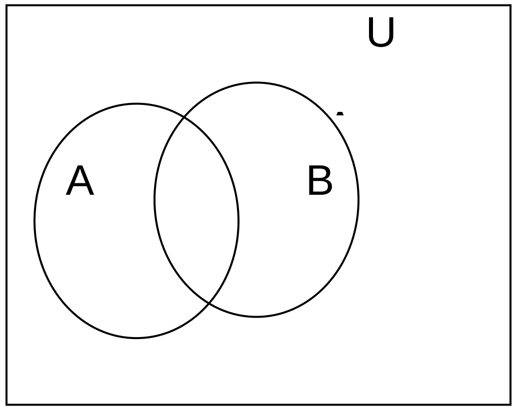
The set of all elements under consideration is called the Universal Set.

The Universal Set is usually denoted by U.

VENN DIAGRAM:

A Venn diagram is a graphical representation of sets by regions in the plane.

The Universal Set is represented by the interior of a rectangle, and the other sets are represented by disks lying within the rectangle.



<u>FINITE AND INFINITE SETS:</u>

A set **S** is said to be **finite** if it contains exactly *m* distinct elements where *m* denotes some non negative integer.

In such case we write $|S| = m$ or $n(S) = m$

A set is said to be **infinite** if it is not finite.

<u>EXAMPLES:</u>

1. The set S of letters of English alphabets is finite and $|S| = 26$
2. The null set ∅ has no elements, is finite and $|\varnothing| = 0$
3. The set of positive integers {1, 2, 3,…} is infinite.

<u>EXERCISE:</u>

Determine which of the following sets are finite/infinite.

1. A = {month in the year} **FINITE**
2. B = {even integers} **INFINITE**
3. C = {positive integers less than 1} **FINITE**
4. D = {animals living on the earth} **FINITE**
5. E = {lines parallel to x-axis} **INFINITE**
6. $F = \{x \in \mathbf{R} \mid x^{100} + 29x^{50} - 1 = 0\}$ **FINITE**
7. G = {circles through origin} **INFINITE**

<u>MEMBERSHIP TABLE:</u>

A table displaying the membership of elements in sets. To indicate that an element is in a set, a 1 is used; to indicate that an element is not in a set, a 0 is used.

Membership tables can be used to prove set identities.

A	A^c
1	0
0	1

The above table is the Member ship table for Complement of A. now in the above table note that if an element is the member of A then it can`t be the member of A^c thus where in the table we have 1 for A in that row we have 0 in A^c.