Semantics and Pragmatics

Lecture No.10

Study Material

Sentence Relations and Truth

Logic and Truth

- Richard Montague (1974), has hypothesized that the tools of logic can help us to represent sentence meaning.
- The study of logic comes down from the Classical Greek world, famously from Aristotle.
- The beginnings of logic lie in a search for the principles of valid argument and inference.

1. Modus Ponens

- A well-known example is Aristotle's Modus Ponens, a type of argument in three steps, for instance,
- A. if Ali left work early, then he is in the sports club.
- B. Ali left work early.
- C. Ali is in the sports club.

$$\boxed{\begin{array}{c} p \rightarrow q \\ p \\ q \end{array}}$$

- If steps 'A' and 'B' (called premises) are true then step 'C' (conclusion) is also guaranteed to be true.
- A horizontal line is used to separate the premises from the conclusion.
- Other rules of valid inference includes:

2. Modus tollens

- 3. Hypothetical syllogism
- 4. Disjunctive syllogism
- Examples are given in the following slides as 2, 3 and 4 respectively.

2. Modus Tollens

- A mode of logical reasoning from a hypothetical proposition according to which if the consequent be denied the antecedent is denied. (as, if A true, B is true; but B is false; therefore A is false)
- In modus Tollens, P implies Q and if the Q is negated then P is also negated.
- Example:

a. if Ali has arrived, then he is in the club.

b. Ali is not in the club.

c. Ali has not arrived.

$$\boxed{\begin{array}{c} p \rightarrow q \\ \neg p \\ \neg q \end{array}}$$

3. Hypothetical syllogism - It is a kind of syllogism which has a conditional statement for one or both of its premises.

- It is also called chain argument because it is based on a transition.
- In hypothetical syllogism $\dfrac{p \rightarrow q,\ q \rightarrow r}{p \rightarrow r}$
- Example:

a. if Ali is in the club, then he is drinking juice.

b. if Ali is drinking juice, then he is drinking Pine apple.

c. If Ali is in the club, then he is drinking Pine apple.

4. Disjunctive syllogism - serving or tending to divide or separate

- It is a valid argument form which is a syllogism having a disjunctive statement for one of its premises
- Only one of the premise can be true (if p is true, then, q is false and vice versa)
- Example 4:

a. Ali is in the club or he is in the lounge.

b. Ali is not in the club.

c. Ali is in the lounge.

$$\boxed{\begin{array}{c} p \vee q \\ \neg p \\ q \end{array}}$$

- A part of this study is a concern for the truth of statements and whether truth is preserved or lost by putting sentences into different patterns.
- Truth here is taken to mean a correspondence with facts, or the correct description of the states of affairs in the world.
- Mostly, truth is said to be empirical as we need some facts of the world to reach it.
- For instance, the truth of the sentence 'My father was the first man to visit Mars' depends on the facts about the life of speaker's father: If her father did go to Mars and was the first man there then the sentence is true; otherwise it is false.

Propositional Logic

- Also known as 'propositional calculus' and 'sentential calculus'.
- Here, we are concerned with the relations between sentences, involving complex sentences, irrespective of the internal structure of the sentences themselves.
- For example, we have two sentences i.e. 'John is in his office' and 'John is at home'.
- Between them, one is true; if the second is false, it can be concluded that the first is true.
- This conclusion can be drawn irrespective of the form of the sentences themselves.
- Semanticists call a sentence's being true or false its truth value, and the facts needed in reality to make a sentence true or false, its truth conditions.
- Moreover, instead of using actual sentences, *p*, *q*, and *r* (sentential variables) can be used to represent the sentences.
- We also need symbols for the logical connectives such as:
- $\neg$ Negation (not)
- $\wedge$ Conjunction (and)
- $\vee$ Disjunction (inclusive 'or')
- $\longrightarrow$ Implication ('if … Then')
- $\equiv$ Equivalence ('if and only if … then')
- A simple example of a linguistic effect on truth value comes from negating a sentence.
- If we have a sentence in English, adding 'not' will reverse the truth value, such as:
- a. your car has been broken.

- b. your car has not been broken.
- If ‘a’ is true then ‘b’ is false; also if ‘a’ is false then ‘b’ is true.
- To show this relationship in logical representation is:
- a. p
- b. ¬ p
- The effect of negation on the truth value of a statement can be shown by a truth table, where ‘T’ represents true and ‘F’ false as below:

p	¬p
T	F
F	T

- This table shows that when p is true (T), ¬p is false (F); when p is false (F), ¬p is true (T).
- This is then a succinct way of describing the truth effect of negation.

Predicate Logic

- The study of the truth behavior of sentences with quantifiers e.g. ‘all, every, each, some, one’ gave rise to predicate logic.
- The propositional logic cannot account for inferences that depends upon such relations within sentences and cannot deal with example such as:
- ‘All men are mortal’, ‘Socrates is a man’, ‘Therefore, Socrates is mortal’.
- For this we need ‘predicate logic’ or ‘predicate calculus’, but as we need to deal with relations between sentences, predicate logic is not wholly distinct from propositional logic, but includes it.
- If we take a simple sentence such as ‘John is a man’ we have a ‘prediction’ in which it is said of the individual John that he has the property of being a man.

It is possible to symbolize this with M(a), where M stands for the predicate ‘is a man’ and (a) refers to the individual ‘John’.

- We can extend this symbolism to deal with relations where more than one individual is concerned.

- Thus ‘John loves Mary’ may be symbolized as L(a, b), where L stands for the predicate ‘loves’ and (a) and (b) for ‘John’ and ‘Mary’.
- It is important to add that the arguments are ‘ordered’.
- since ‘John loves Mary’ (L(a, b)) is not the same as ‘Mary loves John (L(b, a)).
- Other predicates may take even more arguments e.g. ‘John gave Mary a book’ may be shown as G(a, b, c).
- The purpose of this symbolization, however, is to show relations that hold between sentences (or propositions).
- Here, a, b, c are used to refer specific individuals known as ‘individual constants’.
- Similarly, x, y, z can be used to refer to any individual known as ‘individual variables.
- Moreover, introduced the ‘universal quantifier’ ‘∀’ (‘for all’).

Entailment

- Entailment defined by truth – “A sentence p entails a sentence q when the truth of the first (p) guarantees the truth of the second (q), and the falsity of the second (q) guarantees the falsity of the first (p).
- Lets take an example of the sentences below, where a is said to entail b:
- a. The anarchist assassinated the emperor.
- b. The emperor died.
- Assuming that the same individual is denoted by ‘the emperor’.
- If somebody tells us ‘a’ and we believe it, then we know ‘b’ without being told any more.
- It is impossible for somebody to assert ‘a’ but deny ‘b’.
- Entailment is not an inference in the normal sense.
- we do not have to reason to get from ‘a’ to ‘b’, we just know it instantaneously because of our knowledge of language.
- Let’s see how the truth-based definition of entailment, mentioned above, works on this example:
- Step 1: If p (the anarchist assassinated the emperor) is true, is q (the emperor died) automatically true? Yes.

- Step 2: If q (the emperor died) is false, is p (the anarchist assassinated the emperor) also false? Yes.

- Step 3: Then p entails q. Note if p is false then we can’t say anything about q; it can be either true or false.

- To show this relation in logician’s truth tables, we can use the symbols ‘p’ and ‘q’ for our two sentences, and T and F for true and false, but we will add arrows (and) to show the direction of a relation ‘when … then’.

- Composite truth table for entailment is the following:

Logician’s truth table

p		q
T	⇒	T
F	⇒	T or F
F	⇒	F
T or F	⇒	T

- When this set of relations hold between p and q, p entails q.

- From this table we can see that only the truth or the falsity of the entailed sentence have consequences for the other sentence.

Truth Conditional Semantics

- What it is that sentence ‘mean’ under ‘formal semantics’ or ‘truth-conditional semantics’?

- This shares with the logical calculi the basic assumption that sentences are either true or false but is relative to the world.

- It is claimed, to know the meaning of sentence is to know the conditions under which it is true.
- Tarski’s (1956) definition – ‘ a true sentence is one which states that the state of affairs is so and so, and the state of affairs is so and so’.
- For instance, ‘ Snow is white’ if and only snow is white.
- In fact, he proposed this as the basis of a theory of truth, but it is easy to see how it can be converted into a theory of meaning.
- A number of connectives have a predictable effect on the truth value of a compound statement.

- E.g. The truth value of a compound formed by using ‘and’ to join two statements is predictable from the truth of the constituent statement.
- **a.** The house is on fire
- **b.** The fire brigade are on the way
- **c.** The house is on fire and the fire brigade are on the way

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- There are a number of other connectives e.g. if, then and or which can effect the truth value of sentences.
- The logical connectives corresponding to English ‘or’ has two main types: **Inclusive and Exclusive.**
- In **inclusive ‘or’,** the compound is created when one or both the constituents of a sentence are true.
- Example: ‘I will see you today or tomorrow’

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- The **exclusive or** is possible only when one disjunct of the sentence is true and the other is false.
- Example: You will pay the fine or you will go to jail.

p	q	$p \vee_e q$
T	T	F
T	F	T
F	T	T
F	F	F

- The next connective we will look at here is the **material implication**, symbolized as →

- The expression **p** → **q** is only false when **p** (the **antecedent**) is true and **q** (the **consequent**) is false. This connective is something like our use of English *if* . . . *Then*

- ‘If it rains, then I’ll go to the movies’

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

- Much of the work in truth-conditional semantics is of a highly technical nature, and it cannot be rightly evaluated without a detailed exposition.