Digital Logic Design:
An Overview & Number Systems

Analogue versus Digital

Most of the quantities in nature that can be measured are continuous. Examples include

- **Intensity of light during the day:** The intensity of light gradually increases as the sun rises in the morning; it remains constant throughout the day and then gradually decreases as the sun sets until it becomes completely dark. The change in the light throughout the day is gradual and continuous. Even with a sudden change in weather when the sun is obscured by a cloud the fall in the light intensity although very sharp however is still continuous and is not abrupt.
- **Rise and fall in temperature during a 24-hour period:** The temperature also rises and falls with the passage of time during the day and in the night. The change in temperature is never abrupt but gradual and continuous.
- **Velocity of a car travelling from A to B:** The velocity of a car travelling from one city to another varies in a continuous manner. Even if it abruptly accelerates or stops suddenly, the change in velocity seemingly very sudden and abrupt is never abrupt in reality. This can be confirmed by measuring the velocity in short time intervals of few milliseconds.

The measurable values generally change over a continuous range having a minimum and maximum value. The temperature values in a summer month change between 23 ^{0}C to 45 ^{0}C. A car can travel at any velocity between 0 to 120 mph.

Digital representing of quantities

Digital quantities unlike Analogue quantities are not continuous but represent quantities measured at discrete intervals. Consider the continuous signal as shown in the figure 1.1.

To represent this signal digitally the signal is sampled at fixed and equal intervals. The continuous signal is sampled at 15 fixed and equal intervals. Figure 1.2. The set of values (1, 2, 4, 7, 18, 34, 25, 23, 35, 37, 29, 42, 41, 25 and 22) measured at the sampling points represent the continuous signal. The 15 samples do not exactly represent the original signal but only approximate the original continuous signal. This can be confirmed by plotting the 15 sample points. Figure 1.3. The reconstructed signal from the 15 samples has sharp corners and edges in contrast to the original signal that has smooth curves.

If the number of samples that are collected is reduced by half, the reconstructed signal will be very different from the original. The reconstructed signal using 7 samples have missing peak and dip at 34 ^{0}C and 23 ^{0}C respectively. Figure 1.4. The reason for the difference between the original and the reconstructed signal is due to under-sampling. A more accurate representation of the continuous signal is possible if the number of

samples and sampling intervals are increased. If the sampling is increased to infinity the number of values would still be discrete but they would be very close and closely match the actual signal.
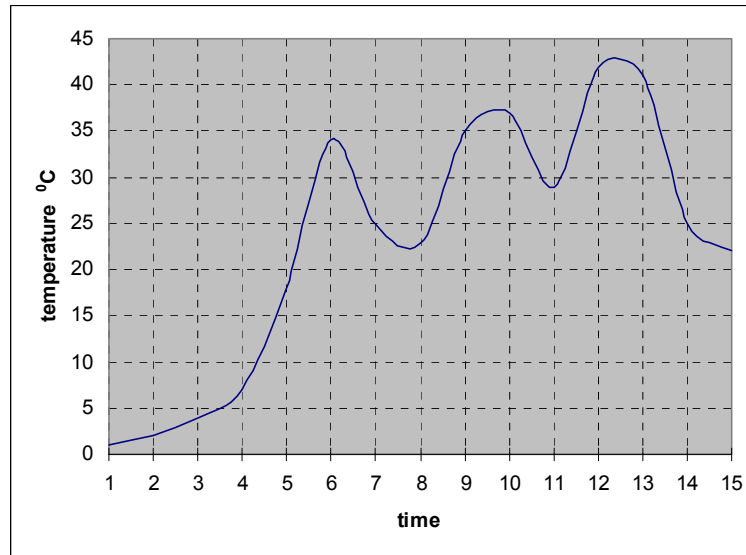
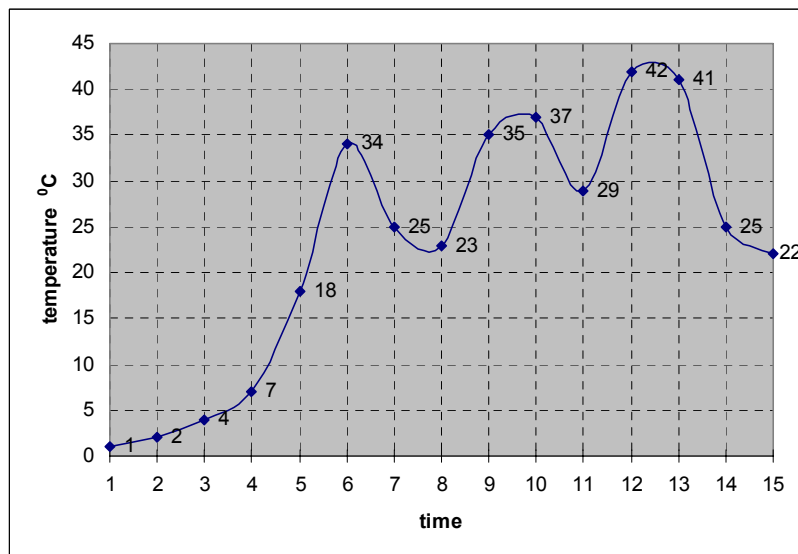Figure 1.1 Continuous signal showing temperature varying with time

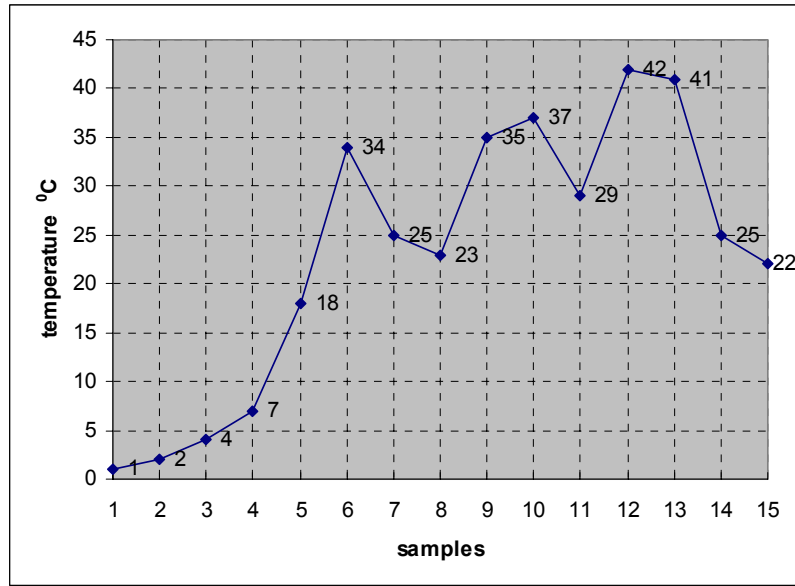Figure 1.2   Sampling the Continuous Signal at 15 equal intervals

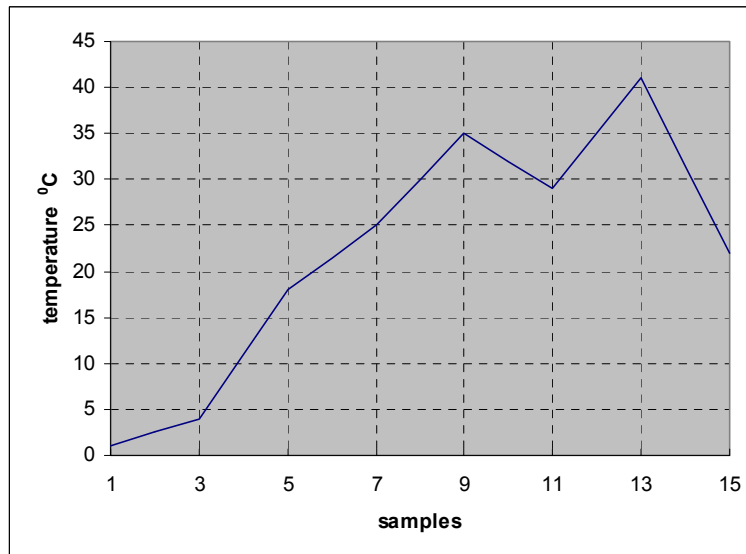Figure 1.3   Reconstructed Signal by plotting 15 sampled values

Figure 1.4   Reconstructed Signal by plotting 7 sampled values

Electronic Processing of Continuous and Digital Quantities

Electronic Processing of the continuous quantities or their Digital representation requires that the continuous signals or the discrete values be converted and represented in terms of voltages. There are basically two types of Electronic Processing Systems.

- **Analogue Electronic Systems:** These systems accept and process continuous signals represented in the form continuous voltage or current signals. The continuous

quantities are converted into continuous voltage or current signals by transducers. The block diagram describes the processing by an Analogue Electronic System. Figure 1.5.

- **Digital Electronic Systems:** These systems accept and process discrete samples representing the actual continuous signal. Analogue to Digital Converters are used to sample the continuous voltage signals representing the original signal.

Do the Digital Electronic Systems use voltages to represent the discrete samples of the continuous signal? This question can be answered by considering a very simple example of a calculator which is a Digital Electronic System. Assume that a calculator is calibrated to represents the number 1 by 1 millivolt (mV). Thus the number 39 is represented by the calculator in terms of voltage as 39 mV. Calculators can also represent large numbers such as 6.25×10^{18} (as in 1 Coulomb = 6.25×10^{18} electrons). The value in terms of volts is 6.25×10^{15} volts! This voltage value can not be practically represented by any electronic circuit. Thus Digital Systems do not use discrete samples represented as voltage values.
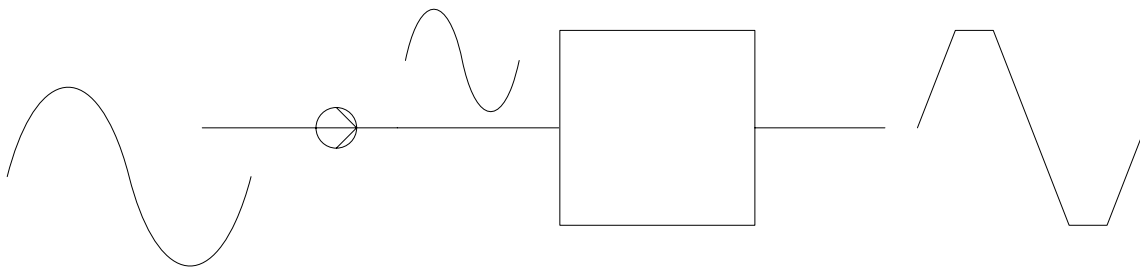

Figure 1.5   Analogue Electronic System processing continuous quantities

Digital Systems and Digital Values

Digital systems are designed to work with two voltage values. A +5 volts represents a logic high state or logic 1 state and 0 volts represents a logic low state or logic 0 state. The Digital Systems which are based on two voltage values or two states can easily represent any two values. For example,

- The numbers '0' and '1'
- The state of a switch 'on' or 'off'
- The colour 'black' and 'white'
- The temperature 'hot' and 'cold'
- An object 'moving' or 'stationary'

Representing two values or two states is not very practical, as many naturally occurring phenomenons have values or state that are more than two. For example, numbers have widely varying ranges, a colour palette might have 64 different shades of the colour red, the temperature of boiling water at room temperature varies from 30 ^{0}C to 100 ^{0}C. Digital Systems are based on the Binary Number system which allows more than two or multiple values to be represented very conveniently.

Binary Number System

The Binary Number System unlike the Decimal number system is based on two values. Each digit or bit in Binary Number system can represent only two values, a '0' and a '1'. A single digit of the Decimal Number system represents 10 values, 0, 1, 2 to 9. The Binary Number System can be used to represent more than two values by combining binary digits or bits. In a Decimal Number System a single digit can represent 10 different values (0 to 9), representing more than 10 values requires a combination of two digits which allows up to 100 values to be represented (0 to 99). A Combination of Binary Numbers is used to represent different quantities.

- **Represent Colours:** A palette of four colours red, blue, green and yellow can be represented by a combination of two digital values 00, 01, 10 and 11 respectively.
- **Representing Temperature:** An analogue value such as 39°C can be represented in a digital format by a combination of 0s and 1s. Thus 39 is 100111 in digital form.

Any quantity such as the intensity of light, temperature, velocity, colour etc. can be represented through digital values. The number of digits (0s and 1s) that represents a quantity is proportional to the range of values that are to be represented. For example, to represent a palette of eight colours a combination of three digits is used. Representing a temperature range of 0^0 C to 100^0 C requires a combination of up to seven digits.

Digital Systems uses the Binary Number System to represent two or multiple values, stores and processes the binary values in terms of 5 volts and 0 volts. Thus the number 39 represented in binary as 100111 is stored electronically in as +5 v, 0v, 0v, +5 v, +5 v and +5 v.

Advantages of working in the Digital Domain

Handling information digitally offers several advantages. Some of the merits of a digital system are spelled out. Details of some these aspects will be discussed and studied in the Digital Logic Design course. Other aspects will be covered in several other courses.

- **Storing and processing data in the digital domain is more efficient:** Computers are very efficient in processing massive amounts of information and data. Computers process information that is represented digitally in the form of Binary Numbers. A Digital CD stores large number of video and audio clips. Sam number of audio and video clips if stored in analogue form will require a number of video and audio cassettes.
- **Transmission of data in the digital form is more efficient and reliable:** Modern information transmission techniques are relying more on digital transmission due to

its reliability as it is less prone to errors. Even if errors occur during the transmission methods exist which allow for quick detection and correction of errors.

- **Detecting and Correcting errors in digital data is easier:** Coding Theory is an area which deals with implementing digital codes that allow for detection and correction of multi-bit errors. In the Digital Logic Design course a simple method to detect single bit errors using the Parity bit will be considered.
- **Data can be easily and precisely reproduced:** The picture quality and the sound quality of digital videos are far more superior to those of analogue videos. The reason being that the digital video stored as digital numbers can be exactly reproduced where as analogue video is stored as a continuous signal can not be reproduced with exact precision.
- **Digital systems are easy to design and implement:** Digital Systems are based on two-state Binary Number System. Consequently the Digital Circuitry is based on the two-voltage states, performing very simple operations. Complex Microprocessors are implemented using simple digital circuits. Several simple Digital Systems will be discussed in the Digital Logic course.
- **Digital circuits occupy small space:** Digital circuits are based on two logical states. Electronic circuitry that implements the two states is very simple. Due to the simplicity of the circuitry it can be easily implemented in a very small area. The PC motherboard having an area of approximately 1 sq.ft has most of the circuitry of a powerful computer. A memory chip small enough to be held in the palm of a hand is able to store an entire collection of books.

Information Processing by a Digital System

A Digital system such as a computer not only handles numbers but all kinds of information.

- **Numbers:** A computer is able to store and process all types of numbers, integers, fractions etc. and is able to perform different kinds of arithmetic operations on the numbers.
- **Text:** A computer in a news reporting room is used to write and edit news reports. A Mathematician uses a computer to write mathematical equations explaining the dissipation of heat by a heat sink. The computer is able to store and process text and symbols.
- **Drawings, Diagrams and Pictures:** A computer can store very conveniently complex engineering drawings and diagrams. It allows real life still pictures or videos to be processed and edited.
- **Music and Sound:** Musicians and Composers uses\ a computer to work on a new compositions. Computers understand spoken commands.

A Digital System (computer) is capable of handling different types of information, which is represented in the form of Binary Numbers. The different types of information use different standards and binary formats. For example, computers use the Binary number system to represent numbers. Characters used in writing text are represented through yet another standard known as ASCII which allows alphabets, punctuation marks and numbers to be represented through a combination of 0s and 1s.

Digital Components and their internal working

Digital system process binary information electronically through specialized circuits designed for handling digital information. These circuits as mentioned earlier operate with two voltage values of +5 volts and 0 volts. These specialized electronic circuits are known as Logic Gates and are considered to be the Basic Building Blocks of any Digital circuit.

The commonly used Logic Gates are the AND gate, the OR gate and the Inverter or NOT Gate. Other gates that are frequently used include NOR, NAND, XOR and XNOR. Each of these gates is designed to perform a unique operation on the input information which is known as a logical or Boolean operation.

Large and complex digital system such as a computer is built using combinations of these basic Logic Gates. These basic building blocks are available in the form of Integrated Circuit or ICs. These gates are implemented using standard CMOS and TTL technologies that determine the operational characteristics of the gates such as the power dissipation, operational voltages (3.3v or 5 v), frequency response etc.
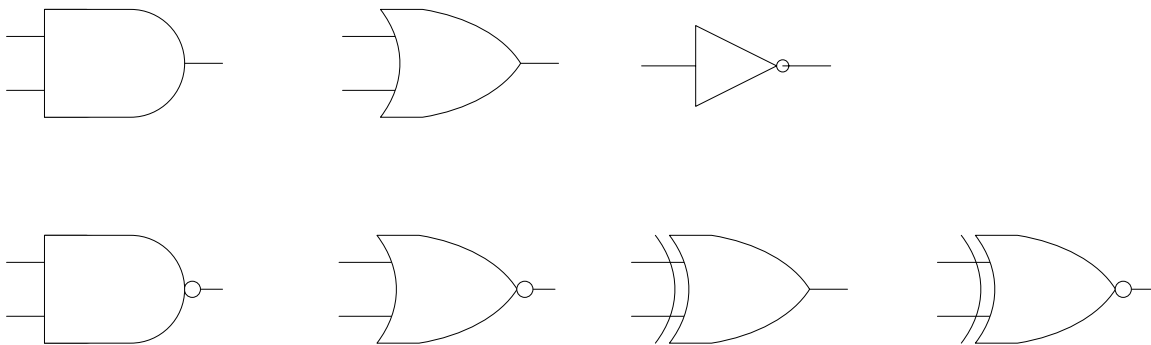

Figure 1.6 Symbolic representations of logic gates.

Combinational Logic Circuits and Functional Devices

The logic gates which form the basic building blocks of a digital system are designed to perform simple logic operations. A single logic gate is not of much use unless it is connected with other gates to collectively act upon the input data. Different gates are combined to build a circuit that is capable of performing some useful operation like adding three numbers. Such circuits are known as Combinational Logic Circuits or Combinational Circuits. An Adder Combinational Circuit that is able to add two single bit binary numbers and give a single bit Sum and Carry output is shown. Figure 1.7.

Implementing large digital system by connecting together logic gates is very tedious and time consuming; the circuit implemented occupies large space, are power hungry, slow and are difficult to troubleshoot.
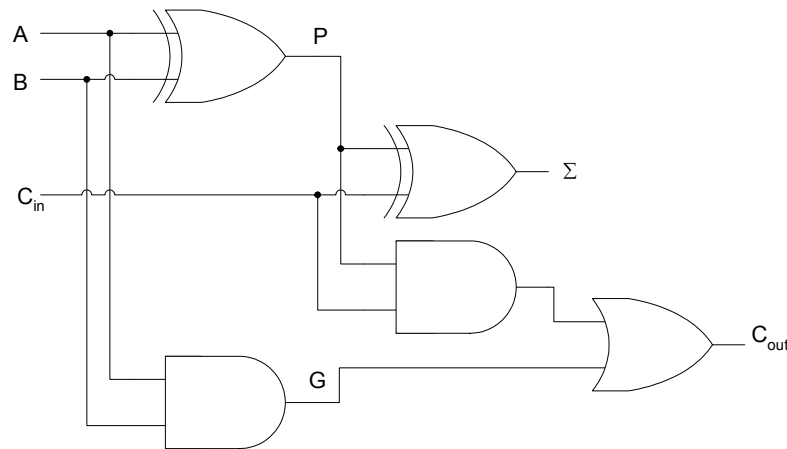
Figure 1.7 1-bit Full-Adder Combinational Circuit

Digital circuits to perform specific functions are available as Integrated Circuits for use. Implementing a Digital system in terms of these dedicated functional units makes the system more economical and reliable. Thus an adder circuit does not have to be implemented by connecting various gates, a standard Adder IC is available that can be readily used. Other commonly used combinational functional devices are Comparators, Decoders, Encoders, Multiplexers and Demultiplexers.

Sequential logic and implementation

Digital systems are used in vast variety of industrial applications and house hold electronic gadgets. Many of these digital circuits generate an output that is not only dependent on the current input but also some previously saved information which is used by the digital circuit. Consider the example of a digital counter which is used by many digital applications where a count value or the time of the day has to be displayed. The digital counter which counts downwards from 10 to 0 is initialized to the value 10. When the counter receives an external signal in the form of a pulse the counter decrements the count value to 9. On receiving successive pulses the counter decrements the currently stored count value by one, until the counter has been decremented to 0. On reaching the count value zero, the counter could switch off a washing machine, a microwave oven or switch on an air-conditioning system.

The counter stores or remembers the previous count value. The new count value is determined by the previously stored count value and the new input which it receives in the form of a pulse (a binary 1). The diagram of the counter circuit is shown in the figure. Figure 1.8.

Digital circuits that generate a new output on the basis of some previously stored information and the new input are known as Sequential circuits. Sequential circuits are a combination of Combinational circuits and a memory element which is able to store some previous information. Sequential circuits are a very important part of digital systems. Most digital systems have sequential logic in addition to the combinational logic. An

example of sequential circuits is counters such as the down-counter which generates a new decremented output value based on the previous stored value and an external input. The storage element or the memory element which is an essential part of a sequential circuit is implemented a flip-flop using a very simple digital circuit known as a flip-flop.
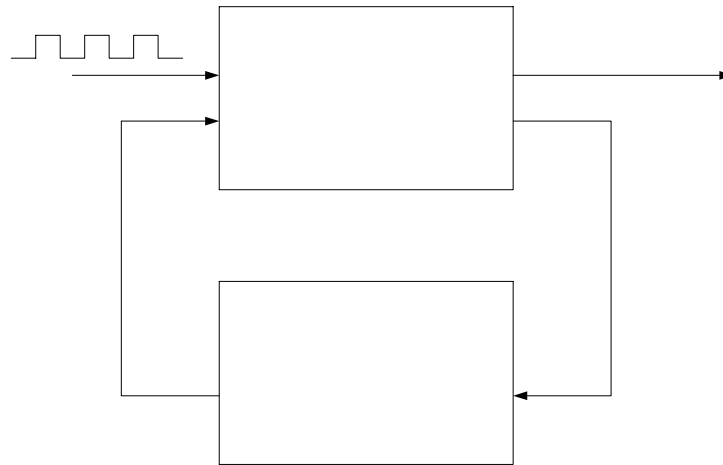

Figure 1.8   A Counter Sequential Circuit

Programmable Logic Devices (PLDs)

The modern trend in implementing specialized dedicated digital systems is through configurable hardware; hardware which can be programmed by the end user. A digital controller for a washing machine can be implemented by connecting together pieces of combinational and sequential functional units. These implementations are reliable however they occupy considerable space. The implementation time also increases. A general purpose circuit that can be programmed to perform a certain task like controlling a washing machine reduces the implementation cost and time.

Cost is incurred on implementing a digital controller for a washing machine which requires that an inventory of all its components such as its logic circuits, functional devices and the counter circuits be maintained. The implementation time is significantly high as all the circuit components have to be placed on a circuit board and connected together. If there is a change in the controller circuit the entire circuit board has to be redesigned. A PLD based washing machine controller does not require a large inventory of components to be maintained. Most of the functionality of the controller circuit is implemented within a single PLD integrated circuit thereby considerably reducing the circuit size. Changes in the controller design can be readily implemented by programming the PLD.

Programmable Logic Devices can be used to implement Combinational and Sequential Digital Circuits.

Memory

Memory plays a very important role in Digital systems. A research article being edited by a scientist on a computer is stored electronically in the digital memory whilst changes are being made to the document. Once the document has be finalized and stored on some media for subsequent printing the memory can be reused to work on some other document. Memory also needs to store information permanently even when the electrical power is turned off. Permanent memories usually contain essential information required for operating the digital system. This important information is provided by the manufacturer of a digital system.

Memory is organized to allow large amounts of data storage and quick access. Memory (ROM) which permanently stores data allows data to be read only. The Memory does not allow writing of data. Volatile memory (RAM) does not store information permanently. If the power supplied to the RAM circuitry is turned off, the contents of the RAM are permanently lost and can not be recovered when power is restored. RAM allows reading and writing of data. Both RAM and ROM are an essential part of a digital system.

Analogue to Digital and Digital to Analogue conversion and Interfacing

Real-world quantities as mention earlier are continuous in nature and have widely varying ranges. Processing of real-world information can be efficiently and reliably done in the digital domain. This requires real-world quantities to be read and converted into equivalent digital values which can be processed by a digital system. In most cases the processed output needs to be converted back into real-world quantities. Thus two conversions are required, one from the real-world to the digital domain and then back from the digital domain to the real-world.

Modern digitally controlled industrial units extensively use Analogue to Digital (A/D) and Digital to Analogue (D/A) converters to covert quantities represented as an analogue voltage into an equivalent digital representation and vice versa. Consider the example of an industrial controller that controls a chemical reaction vessel which is being heated to expedite the chemical reaction. Figure 1.9. Temperature of the vessel is monitored to control the chemical reaction. As the temperature of the vessel rises the heat has to be reduced by a proportional level. An electronic temperature sensor (transducer) converts the temperature into an equivalent voltage value. This voltage value is continuous and proportion to the temperature. The voltage representing the temperature is converted into a digital representation which is fed to a digital controller that generates a digital value corresponding to the desired amount of heat. The digitized output representing the heat is converted back to a voltage value which is continuous and is used to control a valve that regulates the heat. An A/D converter converts the analogue voltage value representing the temperature into a corresponding digital value for processing. A D/A converter converts back the digital heat value to its corresponding continuous value for regulating the heater.
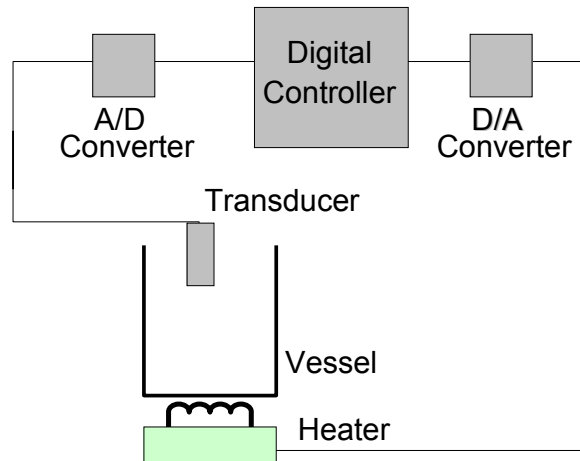
Figure 1.9   Digitally Controlled Industrial Heater Unit

    A/D and D/A converters are an important aspect of digital systems. These devices serve as a bridge between the real and digital world allow the two to communicate and interact together.

Number Systems and Codes

Decimal Number System
    The decimal number system has ten unique digits 0, 1, 2, 3… 9. Using these single digits, ten different values can be represented. Values greater than ten can be represented by using the same digits in different combinations. Thus ten is represented by the number 10, two hundred seventy five is represented by 275 etc. Thus same set of numbers 0,1 2… 9 are repeated in a specific order to represent larger numbers.

    The decimal number system is a positional number system as the position of a digit represents its true magnitude. For example, 2 is less than 7, however 2 in 275 represents 200, whereas 7 represents 70. The left most digit has the highest weight and the right most digit has the lowest weight. 275 can be written in the form of an expression in terms of the base value of the number system and weights.

$$2 \times 10^2 + 7 \times 10^1 + 5 \times 10^0 = 200 + 70 + 5 = 275$$

where, 10 represents the base or radix
        10^2, 10^1, 10^0 represent the weights 100, 10 and 1 of the numbers 2, 7 and 5

Fractions in Decimal Number System
    In a Decimal Number System the fraction part is separated from the Integer part by a decimal point. The Integer part of a number is written on the left hand side of the decimal point. The Fraction part is written on the right side of the decimal point. The digits of the

Integer part on the left hand side of the decimal point have weights 10^0, 10^1, 10^2 etc. respectively starting from the digit to the immediate left of the decimal point and moving away from the decimal point towards the most significant digit on the left hand side. Fractions in decimal number system are also represented in terms of the base value of the number system and weights. The weights of the fraction part are represented by 10^{-1}, 10^{-2}, 10^{-3} etc. The weights decrease by a factor of 10 moving right of the decimal point. The number 382.91 in terms of the base number and weights is represented as

$3 \times 10^2 + 8 \times 10^1 + 2 \times 10^0 + 9 \times 10^{-1} + 1 \times 10^{-2} = 300 + 80 + 2 + 0.9 + 0.01 = 382.91$

Caveman number system

A number system discovered by archaeologists in a prehistoric cave indicates that the caveman used a number system that has 5 distinct shapes Σ, Δ, >, Ω and ↑. Furthermore it has been determined that the symbols Σ to ↑ represents the decimal equivalents 0 to 5 respectively.

Centuries ago a caveman returning after a successful hunting expedition records his successful hunt on the cave wall by carving out the numbers Δ↑. What does the number Δ↑ represent? The table 1.1 indicates that the Caveman numbers Δ↑ represents decimal number 9.

Decimal Number	Caveman Number	Decimal Number	Caveman Number
0	Σ	10	>Σ
1	Δ	11	>Δ
2	>	12	>>
3	Ω	13	>Ω
4	↑	14	>↑
5	ΔΣ	15	ΩΣ
6	ΔΔ	16	ΩΔ
7	Δ>	17	Ω>
8	ΔΩ	18	ΩΩ
9	Δ↑	19	Ω↑
		20	↑Σ

Table 1.1   Decimal equivalents of the Caveman Numbers

The Caveman is using a Base-5 number system. A Base-5 number system has five unique symbols representing numbers 0 to 4. To represent numbers larger than 4, a combination of 2, 3, 4 or more combinations of Caveman numbers are used. Therefore, to represent the decimal number 5, a two number combination of the Caveman number system is used. The most significant digit is Δ which is equivalent to decimal 1. The least significant digit is Σ which is equivalent to decimal 0. The five combinations of Caveman numbers having the most significant digit Δ, represent decimal values 5 to 9 respectively. This is similar to the Decimal Number system, where a 2-digit combination

of numbers is used to represent values greater than 9. The most significant digit is set to 1 and the least significant digit varies from 0 to 9 to represent the next 10 values after the largest single decimal number digit 9.

The Caveman number $\Delta\uparrow$ can be written in expression form based on the Base value 5 and weights 5^0, 5^1, 5^2 etc.

$$= \Delta \text{ x } 5^1 + \uparrow \text{ x } 5^0 = \Delta \text{ x } 5 + \uparrow \text{ x } 1$$

Replacing the Caveman numbers Δ and $\uparrow$ with equivalent decimal values in the expression yields

$$= \Delta \text{ x } 5^1 + \uparrow \text{ x } 5^0 = 1 \text{ x } 5 + 4 \text{ x } 1 = 9$$

The number $\Delta\Omega\uparrow\Sigma$ in decimal is represented in expression form as

$$\Delta \text{ x } 5^3 + \Omega \text{ x } 5^2 + \uparrow \text{ x } 5^1 + \Sigma \text{ x } 5^0 = \Delta \text{ x } 125 + \Omega \text{ x } 25 + \uparrow \text{ x } 5 + \Sigma \text{ x } 1$$

Replacing the Caveman numbers with equivalent decimal values in the expression yields

$$= (1) \text{ x } 125 + (3) \text{ x } 25 + (4) \text{ x } 5 + (0) \text{ x } 1 = 125 + 75 + 20 + 0 = 220$$

Binary Number System

The Caveman Number system is a hypothetical number system introduced to explain that number system other than the Decimal Number system can exist and can be used to represent and count numbers. Digital systems use a Binary number system. Binary as the name indicates is a Base-2 number system having only two numbers 0 and 1. The Binary digit 0 or 1 is known as a 'Bit'. Table 1.2

Decimal Number	Binary Number	Decimal Number	Binary Number
0	0	10	1010
1	1	11	1011
2	10	12	1100
3	11	13	1101
4	100	14	1110
5	101	15	1111
6	110	16	10000
7	111	17	10001
8	1000	18	10010
9	1001	19	10011
		20	10100

Table 1.2   Decimal equivalents of Binary Number System

Counting in Binary Number system is similar to counting in Decimal or Caveman Number systems. In a decimal Number system a value larger than 9 has to be represented by 2, 3, 4 or more digits. In the Caveman Number System a value larger than 4 has to be represented by 2, 3, 4 or more digits of the Caveman Number System. Similarly, in the Binary Number System a Binary number larger than 1 has to be represented by 2, 3, 4 or more binary digits.

Any binary number comprising of Binary 0 and 1 can be easily represented in terms of its decimal equivalent by writing the Binary Number in the form of an expression using the Base value 2 and weights 2^0, 2^1, 2^2 etc.

The number 10011_2 (the subscript 2 indicates that the number is a binary number and not a decimal number ten thousand and eleven) can be rewritten in terms of the expression

$$10011_2 = (1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$
$$= (1 \times 16) + (0 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1)$$
$$= 16 + 0 + 0 + 2 + 1$$
$$= 19$$

Fractions in Binary Number System

In a Decimal number system the Integer part and the Fraction part of a number are separated by a decimal point. In a Binary Number System the Integer part and the Fraction part of a Binary Number can be similarly represented separated by a decimal point. The Binary number 1011.101_2 has an Integer part represented by 1011 and a fraction part 101 separated by a decimal point. The subscript 2 indicates that the number is a binary number and not a decimal number. The Binary number 1011.101_2 can be written in terms of an expression using the Base value 2 and weights 2^3, 2^2, 2^1, 2^0, 2^{-1}, 2^{-2} and 2^{-3}.

$$1011.101_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$$
$$= (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1) + (1 \times 1/2) + (0 \times 1/4) + (1 \times 1/8)$$
$$= 8 + 0 + 2 + 1 + 0.5 + 0 + 0.125$$
$$= 11.625$$

Computers do handle numbers such as 11.625 that have an integer part and a fraction part. However, it does not use the binary representation 1011.101. Such numbers are represented and used in Floating-Point Numbers notation which will be discussed latter.