

Lab Experiment # 04

Boolean algebra and Simplification of Boolean expressions -I

Objectives

Object

- 1- To study DE Morgan's theory and implemented it.
- 2- Learn how to simplify Boolean logic equations using DE Morgan's theory.

DE Morgan's Theory – Background

Augustus De Morgan (27 June 1806 – 18 March 1871) was a British mathematician and logician. He formulated De Morgan's laws.

In simple words, DE Morgan's Theory is used to convert AND/NAND gates to OR/NOR ones, and presented OR/NOR gates by AND/NAND gates by these 2-laws:

$$A + B = (A' \cdot B)'$$

$$A \cdot B = (A' + B)'$$

Basics of Boolean algebra

Boolean Postulates

$$P1: X = 0 \text{ or } X = 1$$

$$P2: 0 \cdot 0 = 0$$

$$P3: 1 + 1 = 1$$

$$P4: 0 + 0 = 0$$

$$P5: 1 \cdot 1 = 1$$

$$P6: 1 \cdot 0 = 0 \cdot 1 = 0$$

$$P7: 1 + 0 = 0 + 1 = 1$$

Boolean Laws

T1: Commutative Law

$$(a) A + B = B + A$$

$$(b) A B = B A$$

T2: Associate Law

$$(a) (A + B) + C = A + (B + C)$$

$$(b) (A B) C = A (B C)$$

T3: Distributive Law

$$(a) A (B + C) = A B + A C$$

$$(b) A + (B C) = (A + B) (A + C)$$

T4: Identity Law

$$(a) A + A = A$$

$$(b) A A = A$$

T5:

$$(a) A B + A \bar{B} = A$$

$$(b) (A+B)(A+\bar{B}) = A$$

T6: Redundancy Law

$$(a) A + A B = A$$

$$(b) A (A + B) = A$$

T7:

$$(a) 0 + A = A$$

$$(b) 0 A = 0$$

T8:

$$(a) I + A = I$$

$$(b) I A = A$$

T9:

$$(a) \bar{A} + A = I$$

$$(b) \bar{A} A = 0$$

T10:

(a) $A + \bar{A} B = A + B$

(b) $A (\bar{A} + B) = AB$

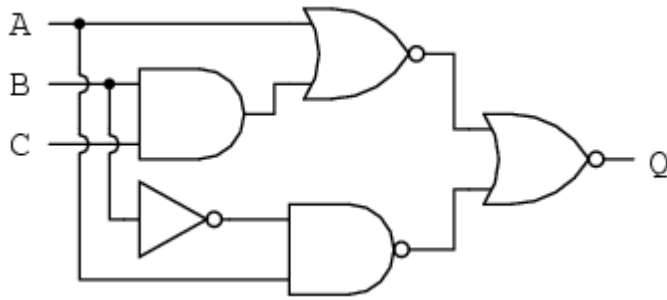
T11: De Morgan's Theorem

(a) $\overline{(A + B)} = \bar{A} \bar{B}$

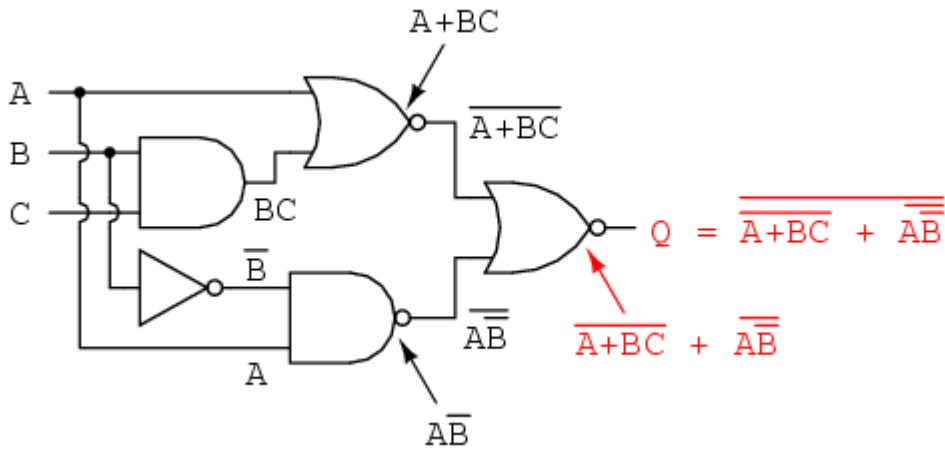
(b) $\overline{A B} = \bar{A} + \bar{B}$

Simplifying Boolean logic functions

Given the following circuit



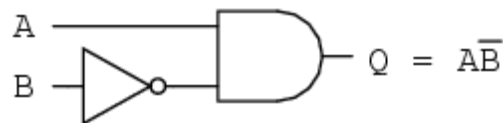
The Boolean expression that represents the above circuit is as follows



We can simplify the above Boolean expression as follows

$$\begin{array}{l}
 \overline{\overline{A + BC + \overline{AB}}} \\
 \downarrow \text{Breaking longest bar} \\
 \overline{\overline{A + BC}} \quad \overline{\overline{AB}} \\
 \downarrow \text{Applying identity } \overline{\overline{A}} = A \text{ wherever double bars of equal length are found} \\
 (A + BC) (\overline{AB}) \\
 \downarrow \text{Distributive property} \\
 A\overline{A}\overline{B} + BC\overline{A}\overline{B} \\
 \downarrow \text{Applying identity } \overline{AA} = A \text{ to left term; applying identity } \overline{AA} = 0 \text{ to B and } \overline{B} \text{ in right term} \\
 \overline{A}\overline{B} + 0 \\
 \downarrow \text{Applying identity } A + 0 = A \\
 \overline{A}\overline{B}
 \end{array}$$

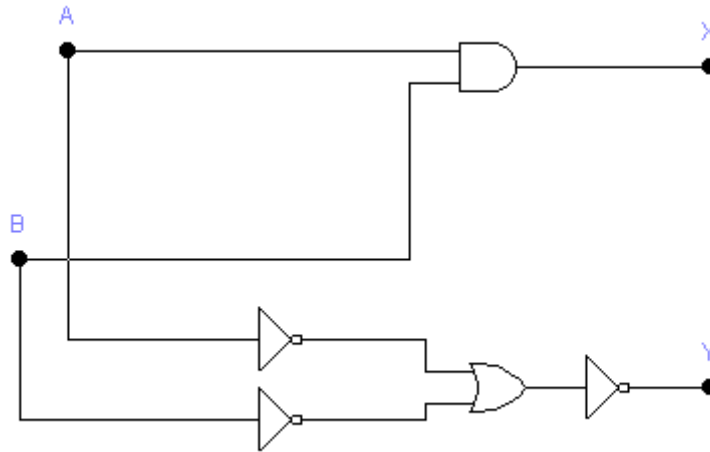
This means that the above circuit can be replaced by the following one



Lab Tasks

Task 1: Circuit analysis

Find the Boolean expression that represents the outputs x and y shown in the following circuit.



According to the circuit above find the equation of X and Y, then fill the truth table.

X =

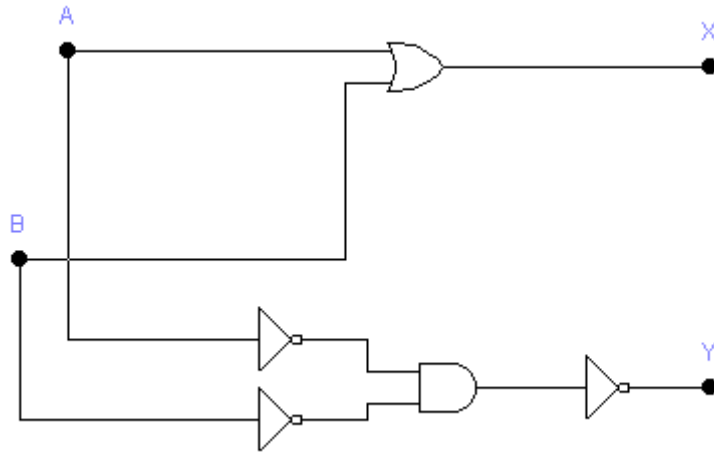
Y =

A	B	X	Y
0	0		
0	1		
1	0		
1	1		

What do you notice?

Task 2: Circuit analysis

Find the Boolean expression that represents the outputs x and y shown in the following circuit.



According to the circuit above find the equation for X and Y, then fill the truth table.

X =

Y =

A	B	X	Y
0	0		
0	1		
1	0		
1	1		

What do you notice?

Task 3: Simplifying Boolean functions

Simplify the following Boolean expression

$$F(A, B) = (A \cdot B) + A' (A+B)$$

Draw the simplified and the original Boolean expression using EWB and make sure that they are both equivalent by filling-in the following truth table.

A	B	F (A, B) (original)	Y (Simplified)
0	0		
0	1		
1	0		
1	1		

Task 4: Simplifying Boolean functions

Simplify the following Boolean expression



$$F(A, B, C) = (A+C') + C (C \cdot A' + (B \cdot A) + C)$$

Draw the simplified Boolean expression using EWB. Find out the truth table of the circuit.


Task 5: Simplifying Boolean functions in EWB using the logic converter

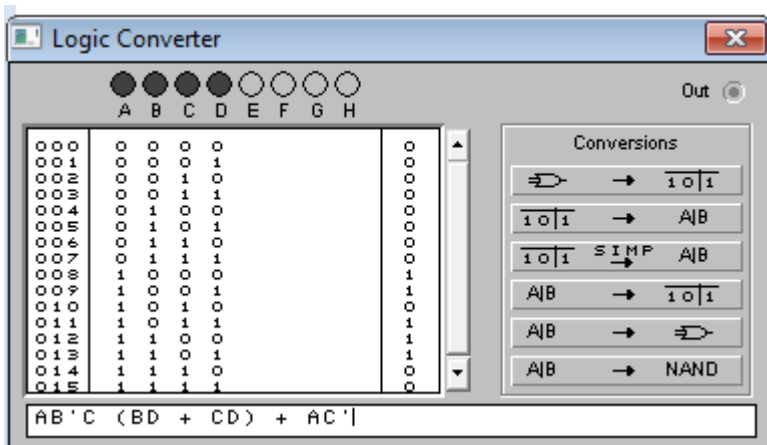
Simplify the following Boolean expression in EWB using the logic converter

$$F(A, B, C) = AB'C(BD + CD) + AC'$$

To do so, you need to enter the expression as shown below, and then click on the following button  to extract the truth table of the expression. Finally, click on the following button  that will generate the simplified form of the equation.

To draw the circuit after simplification, you need to click on the following button

, this will realize the simplified expression using basic gates.



Task 6: Simplifying Boolean functions in EWB using the logic converter

Simplify the following Boolean expression in EWB using the logic converter

$$F(A, B, C) = AB'C + A'B'C + A'BC' + A'B'C$$